Unraveling nonlinear plasma dynamics in electric propulsion with data-driven analysis techniques

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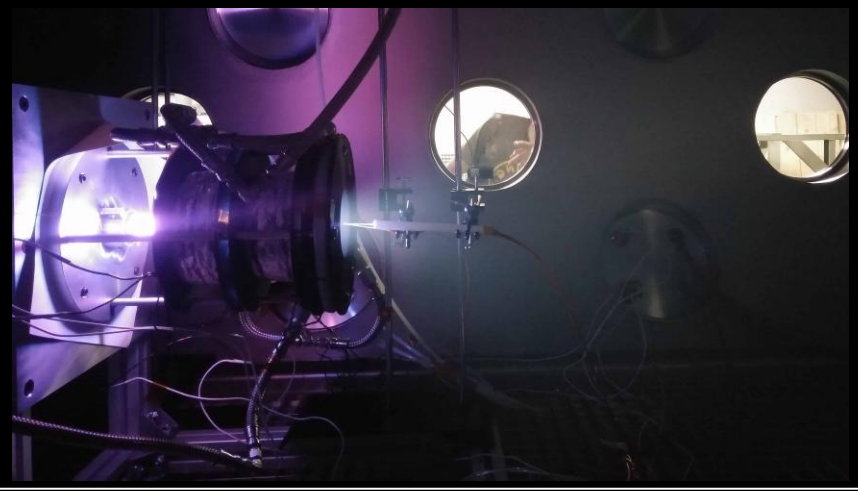
ZARATHUSTRA EP² uc3m etc

Nonlinear plasma couplings

- Space plasma thrusters offer a much higher propellant efficiency than chemical rockets; >50% satellites
- Magnetized plasmas in space thrusters feature a wide array of oscillations
  - Some are applied (e.g. heating EM fields), yet others arise from instabilities
  - Nonlinear dynamics are...
    - ...essential for the saturation of instabilities and to transfer power across modes
    - ...a key ingredient for the development of (essentially-2D) turbulence, which enhances plasma transport and losses to the walls

> To what extent can we address nonlinear plasma dynamics in these thrusters using the formalism of weak turbulence and higher-order spectral analysis?



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- Review of High-order Spectral analysis tools (i.e. bispectrum and friends)

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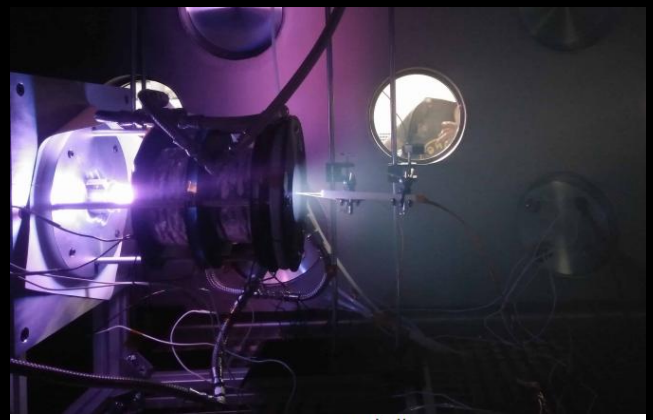

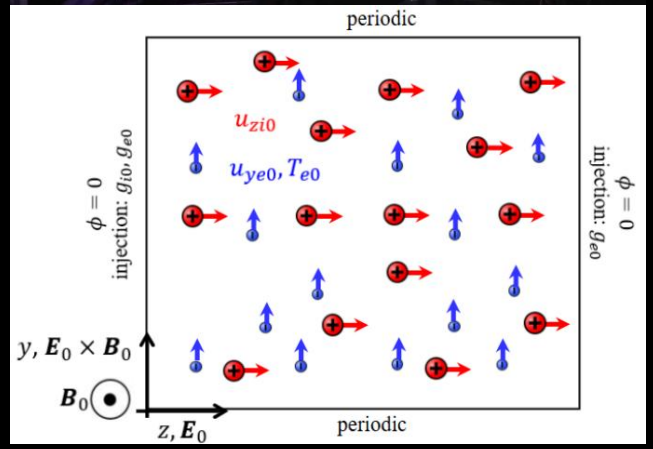

Review of High-order Spectral analysis

- We will discuss signals $x_a(t, y)$ (resolved in time and one spatial coordinate
- Discrete Fourier transform:

$$x_a(t_i, y_j) = \sum_m \hat{x}_a(\omega_m, y_j) e^{-\mathrm{i}\omega_m t_i} = \sum_n \tilde{x}_a(t_i, k_n) e^{\mathrm{i}k_n y_j} = \sum_n \sum_m \hat{\tilde{x}}_a(\omega_m, k_n) e^{-\mathrm{i}\omega_m t_i + \mathrm{i}k_n y_j}$$

- (Cross-) Power Spectal Density (PSD) estimation

$$P_{ab}(\omega_m, k_n) = \left\langle \hat{\tilde{x}}_a(\omega_m, k_n) \hat{\tilde{x}}_b^*(\omega_m, k_n) \right\rangle$$

  - Large CSD among two signals x_a, x_b relates to consistent phase difference over realizations – traveling waves across points a, b

- (Cross-) Bispectrum estimation and bicoherence (i.e., normalized bispectrum)

$$\mathfrak{B}_{abc}(\omega_m, \omega_l, k_n, k_p) = \left\langle \hat{\tilde{x}}_a(\omega_m, k_n) \hat{\tilde{x}}_b(\omega_l, k_p) \hat{\tilde{x}}_c^*(\omega_m + \omega_l, k_n + k_p) \right\rangle$$

$$b_{abc}(\omega_m, \omega_l, k_n, k_p) = \frac{|\mathfrak{B}_{abc}(\omega_m, \omega_l, k_n, k_p)|}{\sqrt{\left\langle \left| \hat{\tilde{x}}_a(\omega_m, k_n) \hat{\tilde{x}}_b(\omega_l, k_p) \right|^2 \right\rangle \left\langle \left| \hat{\tilde{x}}_c^*(\omega_m + \omega_l, k_n + k_p) \right|^2 \right\rangle}}$$

  - Bicoherence measures the consistency of phase lock among modes $m, l, m + l$, which relates to a quadratic coupling. It is $\simeq 1$ for good phase lock and $\simeq 0$ for random, unrelated phases.
  - There are many symmetries: $b_a(\omega_m, \omega_l) = b_a(\omega_l, \omega_m) = b_a(\omega_m, -\omega_m - \omega_l) = b_a(-\omega_m, -\omega_l)$

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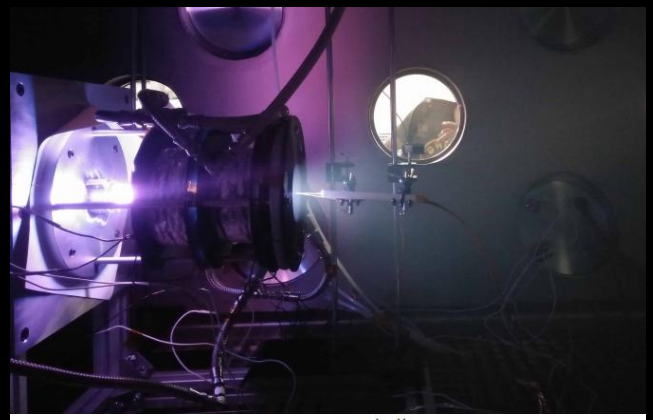
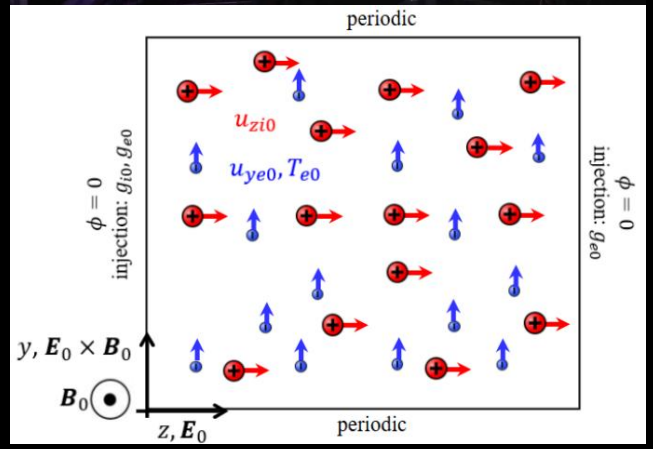
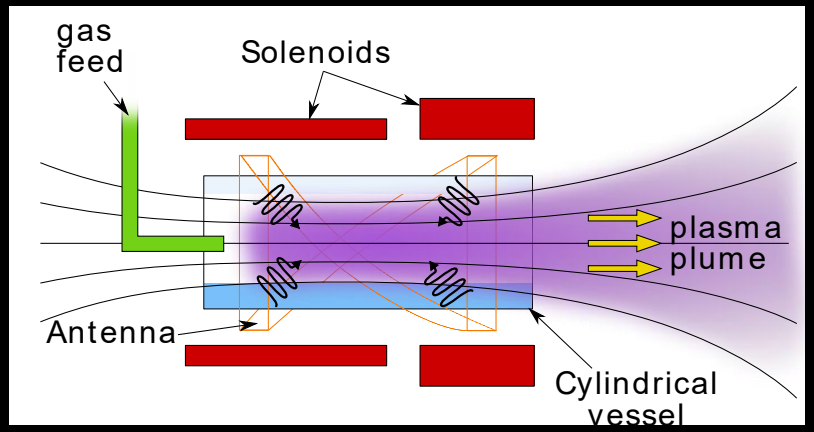
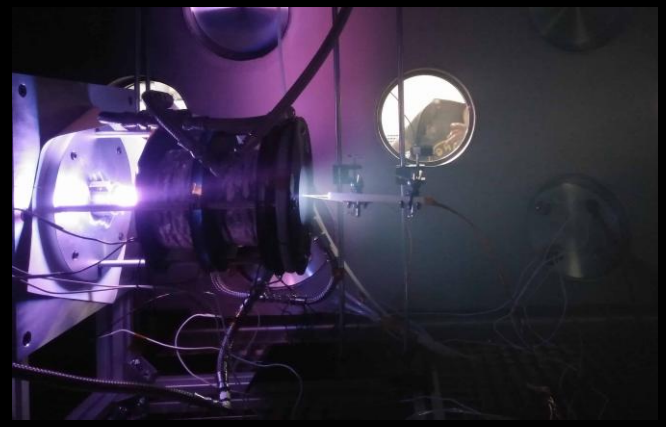


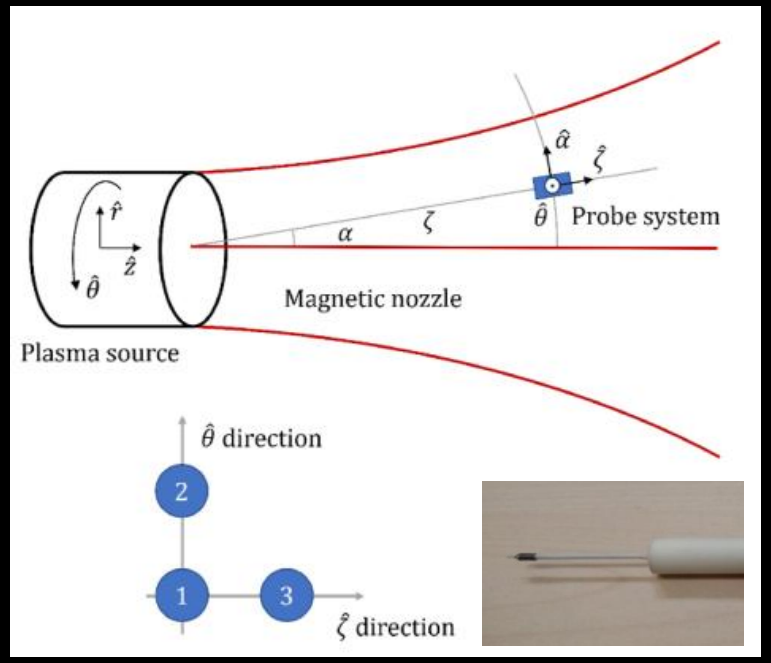


Helicon Plasma Thruster (HPT) experiment

- A Helicon Plasma Thruster consist of a dielectric discharge chamber, an inductor a gas feed, and an applied magnetic field. Our device operates @ 13.56 MHz, <1 kW, 0-50 sccm Xe (can also use other gases)
- Applied field...
  - ...*Enables* the propagation of waves (whistler modes) inside the dense plasma, which heat the electrons
  - ...Creates an external magnetic nozzle for efficient and contactless plasma expansion and magnetic thrust generation
- We set up an array of 3 probes in the plume (separated 1 cm wrt each other)
  - Probes measure time-resolved floating potential ϕ_p simultaneously at 3 locations
  - We scan the plume with a robotic arm

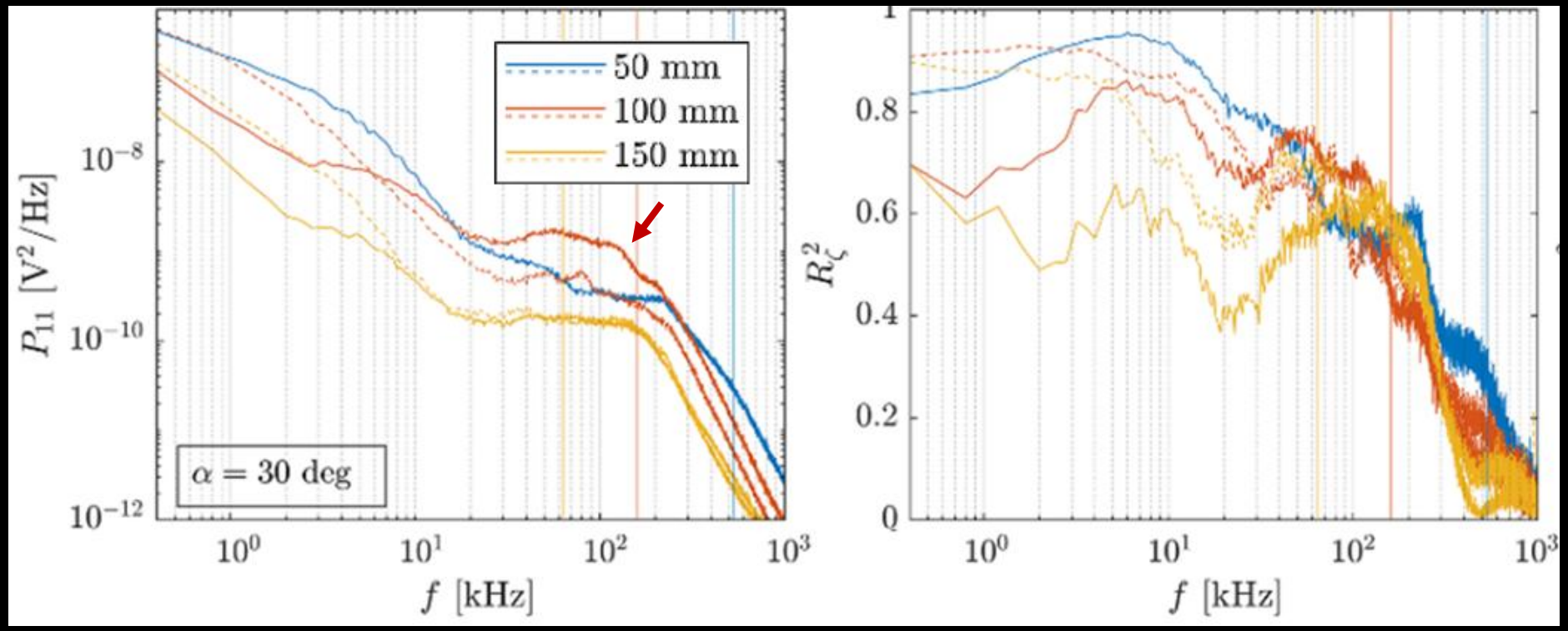






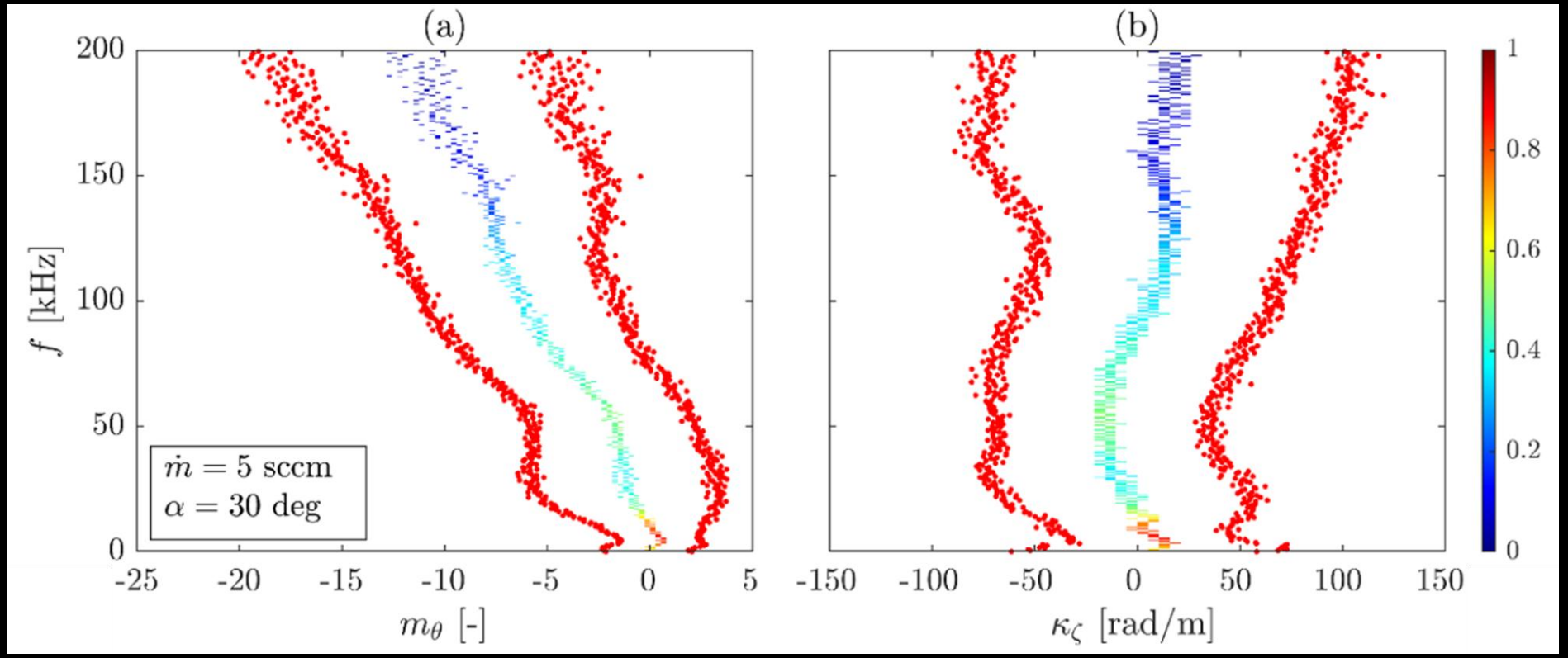
Characterization of low-frequency oscillations

- PSD shows important low-frequency wideband oscillations up to ~200 kHz
  - In the 20-200 kHz range they are larger at 100 mm from the source than at 50 mm
- Coherence (normalized CSD, $R^2 = |P_{12}|^2/P_{11}P_{22}$) is high up to ~ 200 kHz
- This suggests traveling wave structures exist in the azimuthal direction of the plasma plume



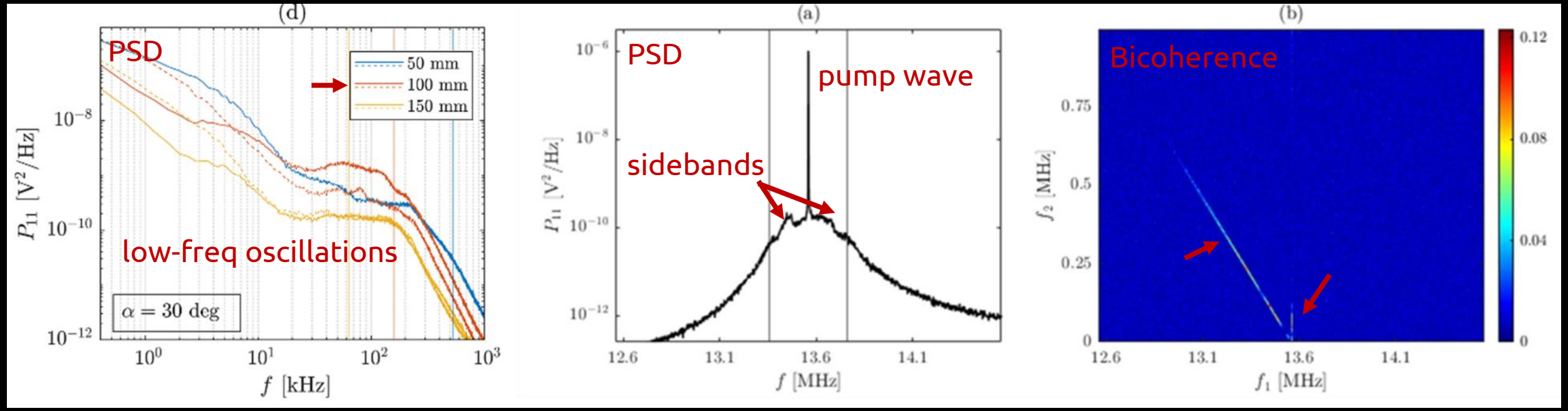
Characterization of low-frequency oscillations

- If we use the phase angle of the CSD and divide by the distance between probes (1 cm), we can estimate the dispersion diagram of the waves. We find essentially-azimuthally propagating modes
- The diagram differs at other locations – complex jumps suggest more than one wave is present at each f



Evidence of nonlinear coupling with the pump wave

- Bicoherence among [0-200 kHz], [13.36-13.56 MHz], [13.56 MHz] is statistically significant
- Most likely explanation: nonlinear power coupling associated to 'parametric decay instability', wherein the pump wave power decays into the low frequency spectrum and the high-frequency sideband
  - Also reported in many helicon plasma sources for manufacturing
  - Believed to be an important mechanism in electron heating



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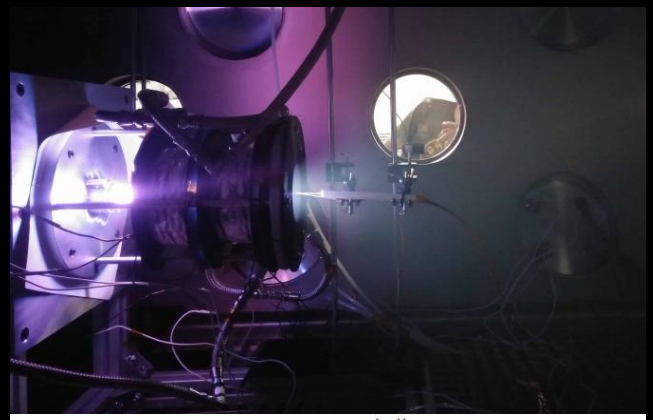
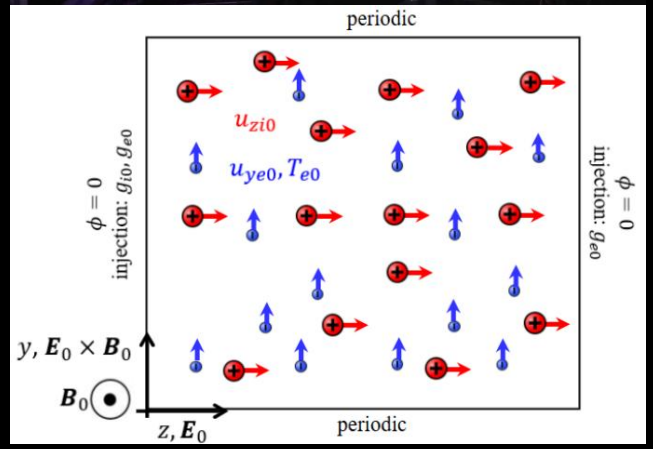
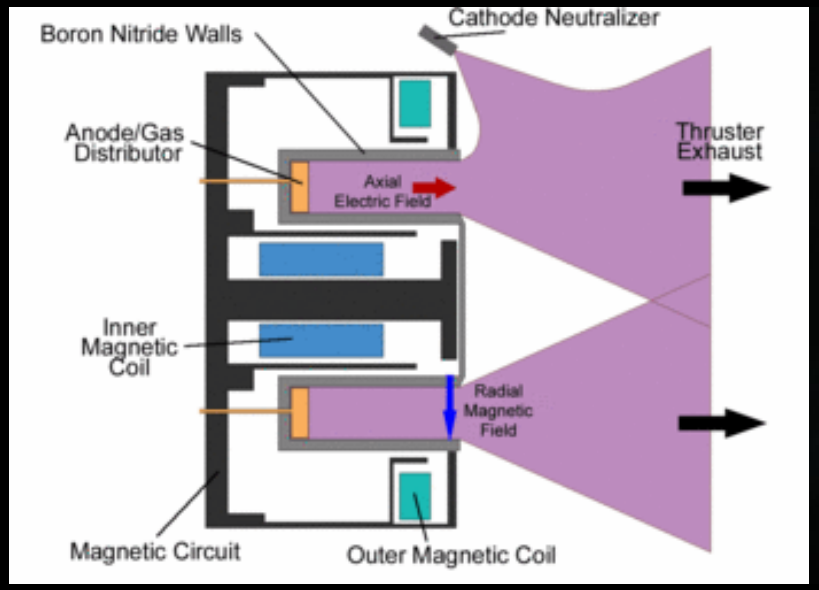
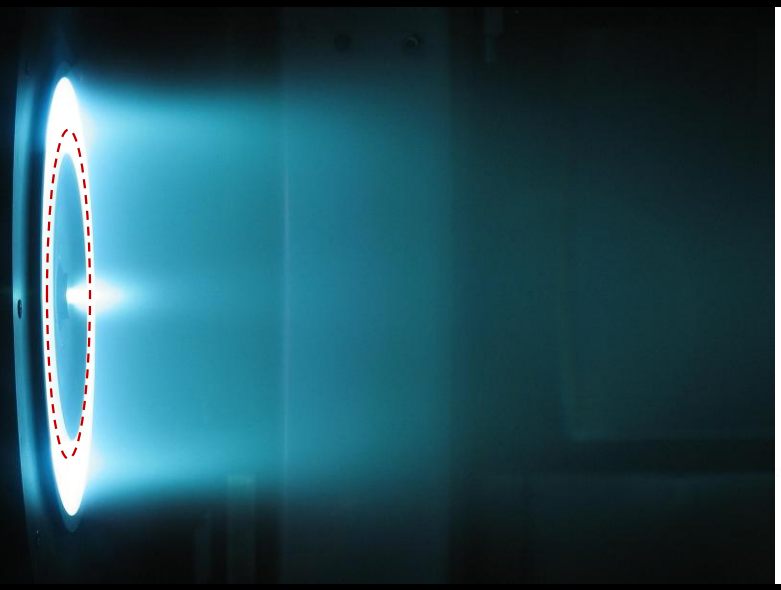


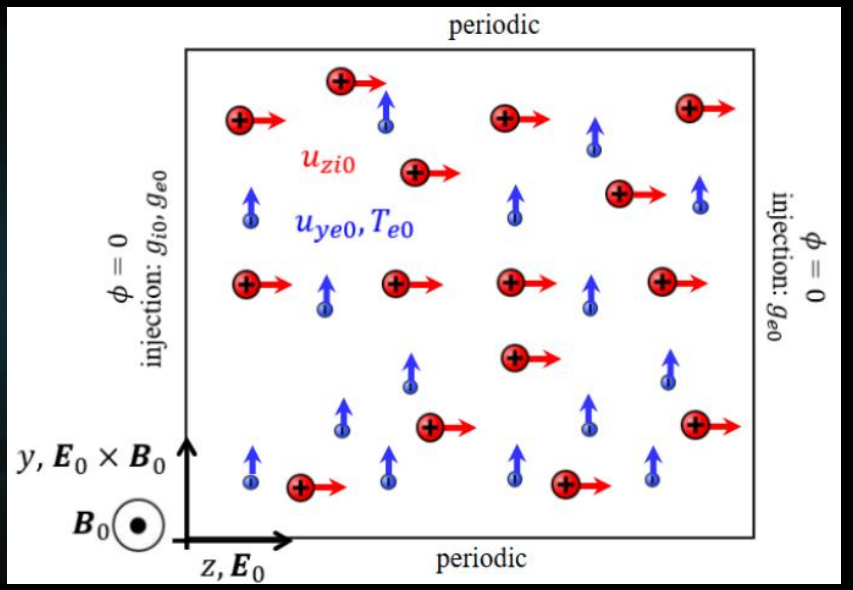


Hall effect thruster (HET) particle-in-cell simulations

- Lagrangian+Eulerian (particle-in-cell) simulation of the ion-electron plasma inside a HET discharge channel
  - Axial-azimuthal domain; collisionless simulation
  - Radial (i.e. out of plane) applied B field
  - Axial applied E field, plus self-consistent E field
  - Injection of thermal electrons at both ends; injection of cold, streaming ions from the left



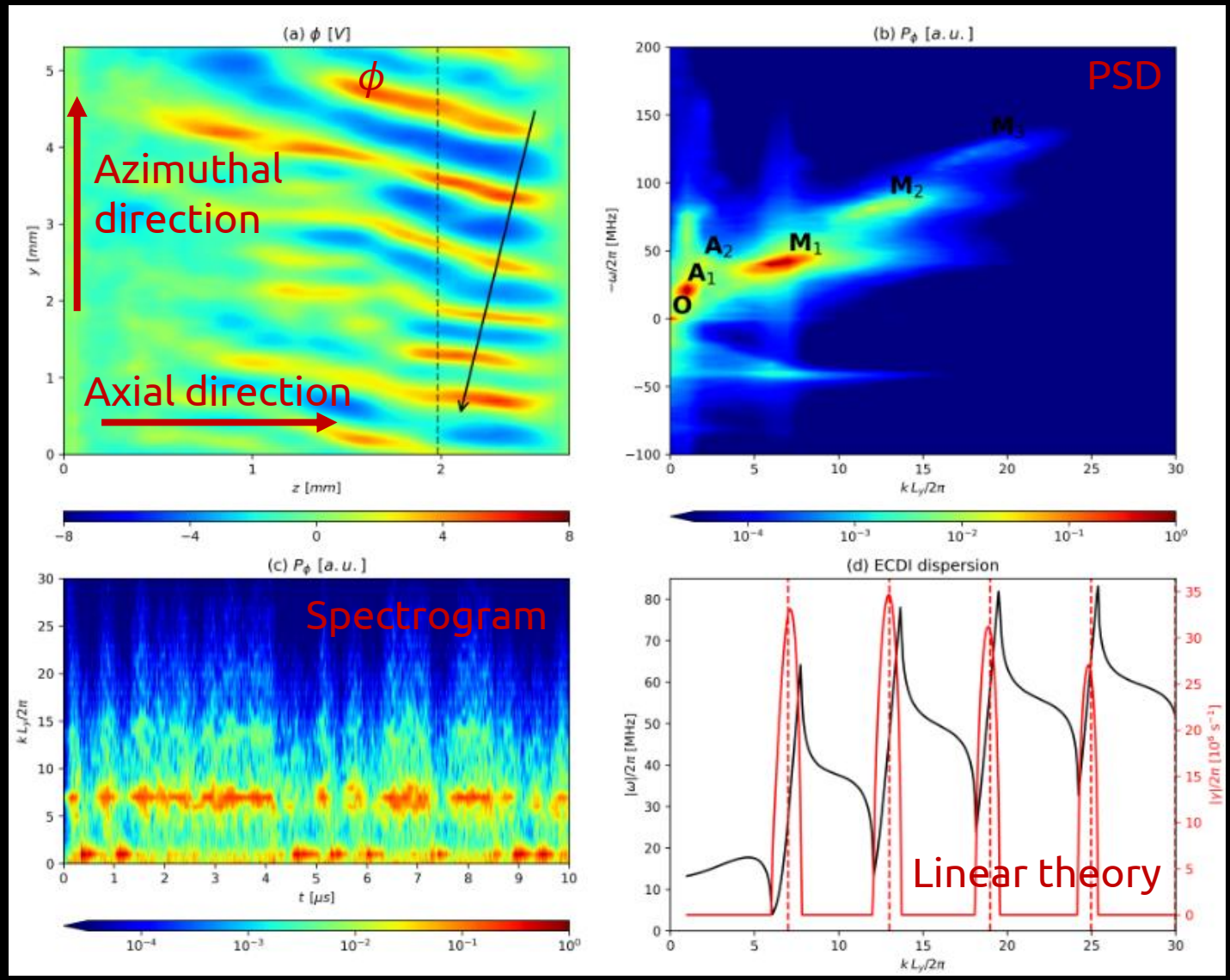




Oscillations and effect on axial electron transport

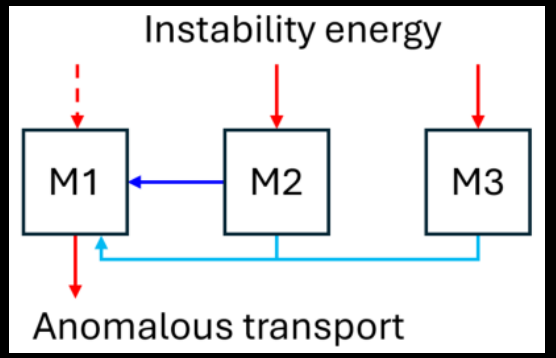
- Self-organized oscillations occur that are essentially azimuthal
- Wavenumbers match with the modes of electron-cyclotron drift instability (ECDI) (linear theory)
- Additional low-frquency modes exist
- Time evolution shows alternation among low frequency modes and ECDI modes

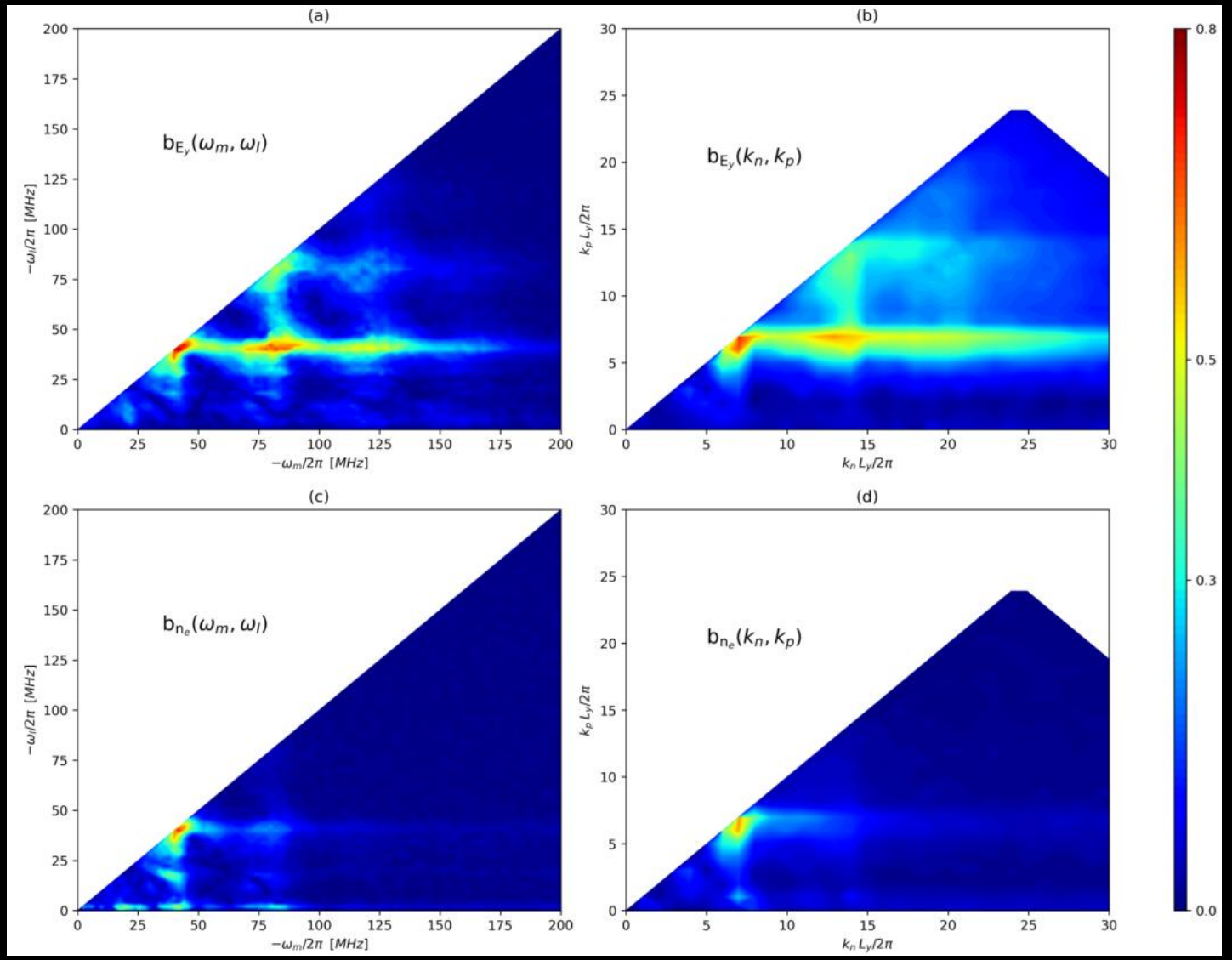
- The presence of correlated oscillations in ϕ and n_e causes an axial $E \times B$ transport of electrons (which is zero in their absence)
  - Largest contributor to transport is M1
- This is wave-induced (turbulent?) anomalous transport that reduces the efficiency of the device



Evidence of nonlinear coupling among modes

- Bicoherence of electric field E_y reveals strong coupling among M1, M2 , M3 ECDI modes
  - The coupling is less evident in the bicoherence diagrams for n_e
- Symbolic regression model suggests nonlinear couplings make power flow from M3 and M2 to M1
- Most likely explanation? inverse energy cascade M3 – M2 – M1 – anomalous transport
  - Conclusion in agreement with other studies





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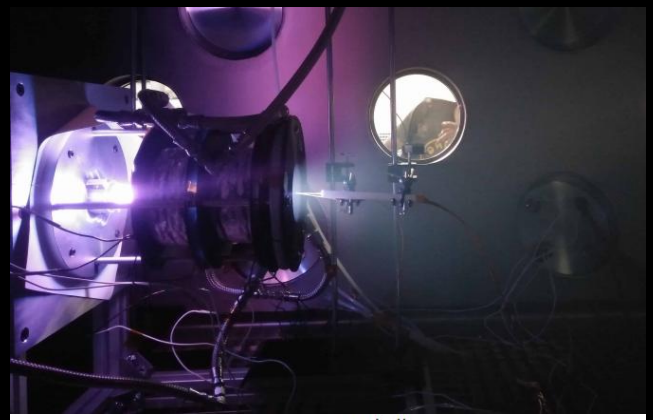

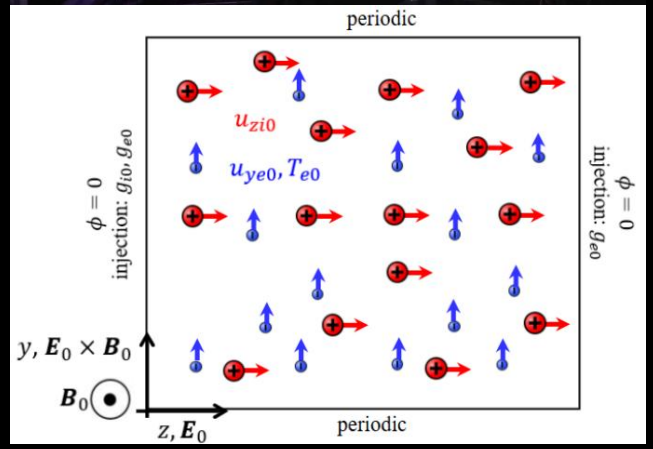

Conclusions and Wrap-up

- Plasmas in space thrusters exhibit a wide range of oscillations, some of them applied, others self-induced
- Nonlinear dynamics are an essential saturation mechanism for instabilities, transferring power among modes, and participate in the creation of anomalous transport
- Bispectrum and related tools are useful to explore these physics in electric propulsion (on both experimental and numerical datasets), at least when the situation can be described as mode-dominated weak turbulence
  - What if this is not the case?
  - What other tools can be relevant?

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Thank you!

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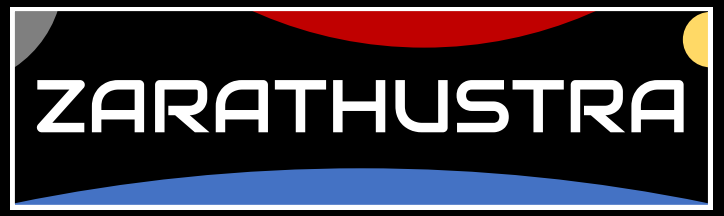



Simulation setup parameters

Description and symbol	Value and units
Ion mass, m_i	1 amu
Applied electric field, E_0	10^4 V/m
Applied magnetic field, B_0	200 G
Plasma density, n_0	10^{17} m^{-3}
Ion axial velocity, u_{zi0}	10 km/s
Electron temperature, T_{e0}	6 eV
Azimuthal domain length, L_y	5.359 mm
Axial domain length, L_z	2.679 mm
$E_0 \times B_0$ drift, u_{ye0}	500 km/s
Electron thermal speed, c_{e0}	1027 km/s
Ion sound speed, c_{s0}	23.97 km/s
Debye length, λ_{D0}	57.58 μm
Electron Larmor radius, ρ_{e0}	292.0 μm
Electron plasma frequency, ω_{pe0}	2.839 GHz
Electron gyrofrequency, ω_{ce}	0.5600 GHz
Ion plasma frequency, ω_{pi0}	66.26 MHz
Lower-hybrid frequency, ω_{lh}	13.07 MHz
Number of cells in y direction, N_y	100
Number of cells in z direction, N_z	50
Number of particles per cell, N_{ppc}	~ 200
Time step, Δt	$5 \cdot 10^{-12}$ s
Print-out time step, Δt_{print}	10^{-9} s
Total simulation time, t_{final}	$50 \cdot 10^{-6}$ s
Cell size, Δy, Δz	53.59 μm

SINDy regression models

	ε_i^S	Model
1	0.99	$\partial P_1/\partial t = 1.25 \cdot 10^{-3}\sqrt{P_1 P_1 P_2}$
2	**0.88**	$\boldsymbol{\partial P_1/\partial t = -8.72 \cdot 10^{5} P_1 + 8.90 \cdot 10^{-3}\sqrt{P_1 P_1 P_2}}$
3	0.87	$\partial P_1/\partial t = -9.2 \cdot 10^{5} P_1 + 9.06 \cdot 10^{-3}\sqrt{P_1 P_1 P_2} - 9.10 \cdot 10^{1}\partial P_1/\partial z$
4	0.87	$\partial P_1/\partial t = -8.22 \cdot 10^{5} P_1 + 6.40 \cdot 10^{-3}\sqrt{P_1 P_1 P_2} + 4.02 \cdot 10^{-2}\sqrt{P_1 P_2 P_3} - 1.14 \cdot 10^{2}\partial P_1/\partial z$
1	0.97	$\partial P_2/\partial t = -6.18 \cdot 10^{-5}\sqrt{P_1 P_1 P_2}$
2	**0.93**	$\boldsymbol{\partial P_2/\partial t = 5.94 \cdot 10^{5} P_2 - 1.66 \cdot 10^{-4}\sqrt{P_1 P_1 P_2}}$
3	0.93	$\partial P_2/\partial t = 6.20 \cdot 10^{5} P_2 - 1.63 \cdot 10^{-4}\sqrt{P_1 P_1 P_2} - 2.68 \cdot 10^{-3}\sqrt{P_1 P_2 P_3}$
4	0.93	$\partial P_2/\partial t = 6.34 \cdot 10^{5} P_2 - 1.66 \cdot 10^{-4}\sqrt{P_1 P_1 P_2} - 2.48 \cdot 10^{-3}\sqrt{P_1 P_2 P_3} - 1.01 \cdot 10^{1}\partial P_2/\partial z$
1	0.99	$\partial P_3/\partial t = -4.72 \cdot 10^{-5}\sqrt{P_1 P_2 P_3}$
2	**0.96**	$\boldsymbol{\partial P_3/\partial t = 7.00 \cdot 10^{5} P_3 - 1.89 \cdot 10^{-4}\sqrt{P_1 P_2 P_3}}$
3	0.94	$\partial P_3/\partial t = 6.62 \cdot 10^{5} P_3 - 1.84 \cdot 10^{-4}\sqrt{P_1 P_2 P_3} - 4.64 \cdot 10^{1}\partial P_3/\partial z$