Advanced 2D fluid and PIC approaches to model ExB plasma instabilities

A. Marín-Cebrián E. Ahedo, M. Merino, P. Fajardo,
D. Poli, M. Ripoli, G. Cuerva-Lázaro, M. P. Encinar
Department of Aerospace Engineering, Universidad Carlos III de Madrid

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Outline

- Introduction
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    - 2D GPU PIC model of the acceleration region of a HET
    - PIC formulation with modal decomposition: application to a Penning discharge
  - Fluid formulation
    - 5-moment model
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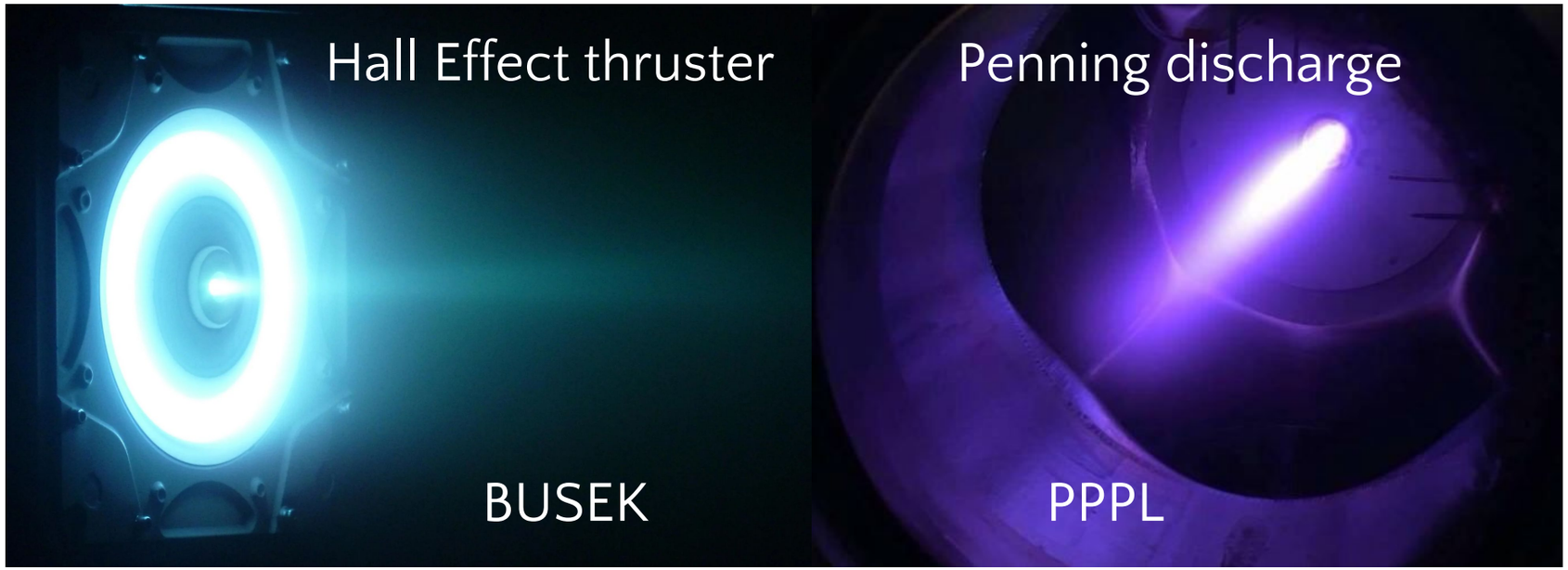
    - Application to HET and plasma column scenarios
- Conclusions and future work

Introduction

- ExB plasmas
  - Mutually perpendicular electric and magnetic fields
  - Increased residence time of magnetized electrons
  - First observation of **E**x**B** discharges: C. E. S. Phillips and W. Crookes. Proc. R. Soc. 64, 172 (1899)

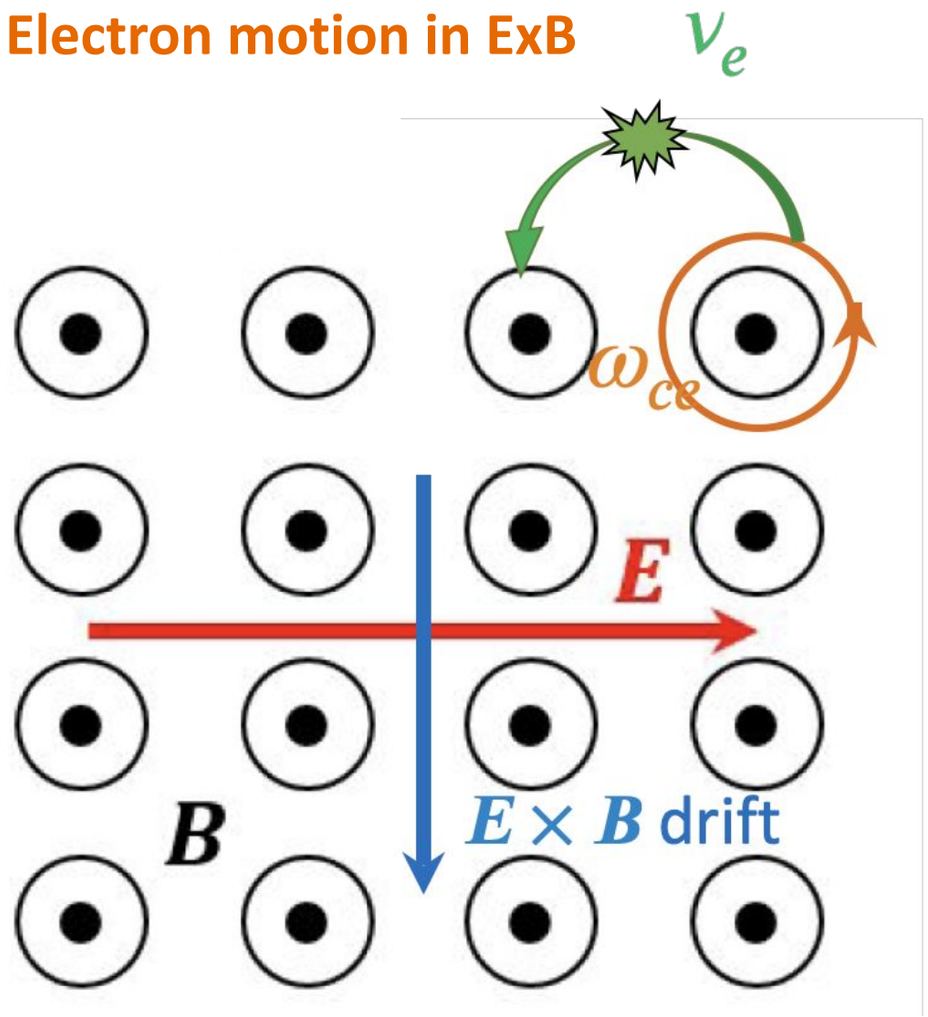
- Common ExB sources



Plasma instabilities leading to “anomalous” cross field electron transport.

Anomalous cross field electron transport

Electron motion in ExB



Motion of a single electron particle in **ExB** configurations.

- Free motion along magnetic field lines.
- Gyromotion around magnetic field lines.
- Drift in the ExB direction: $\mathbf{u}_{ExB} = \mathbf{E}\times\mathbf{B}/B^2$
- Cross-field drift due to…

Collisions?

Experimental data show electron cross-field current that cannot be explained only by the action of classical collisions.

The enhanced cross field electron transport is mainly attributed to correlated oscillations in plasma density and electric field in the **ExB** direction driven by plasma instabilities.

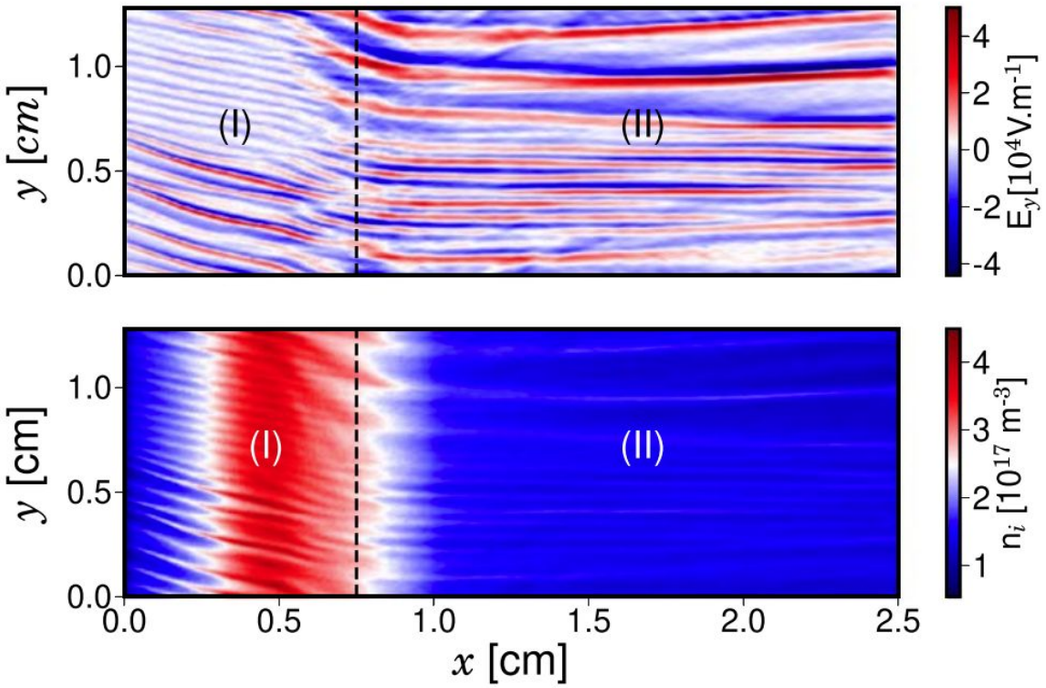
Instabilities in Hall effect thrusters

- Many different oscillations have been observed in HETs
  - Breathing mode
    - frequency in the order of 10 kHz
  - Ion transit time
    - frequency in the order of 100kHz
  - High frequency waves
    - frequency in the order of MHz
    - Observed experimentally
      S. Tsikata et al. Phys. Plasmas. 16, 033506 (2009)
    - It is believed to be the main contributor to anomalous transport in HETs
    - They are considered to be driven by the Electron Cyclotron Drift Instability (ECDI).
      D. Forslund et al. Phys. Rev. Lett. 25, 1266 (1970)

- The simulation of anomalous transport in HETs has typically relied on particle-in-cell (PIC) models.
  - 1D models e.g. T. Lafleur et al. Phys. Plasmas 23, 053502 (2016)
  - 2D models e.g. J. Adam et al. Phys. Plasmas 11, 295 (2004)
  - 3D models e.g. W. Villafana et al. Phys. Plasmas 30, 033503 (2023)

PIC code benchmark based on a HET-like configuration showing azimuthal plasma waves

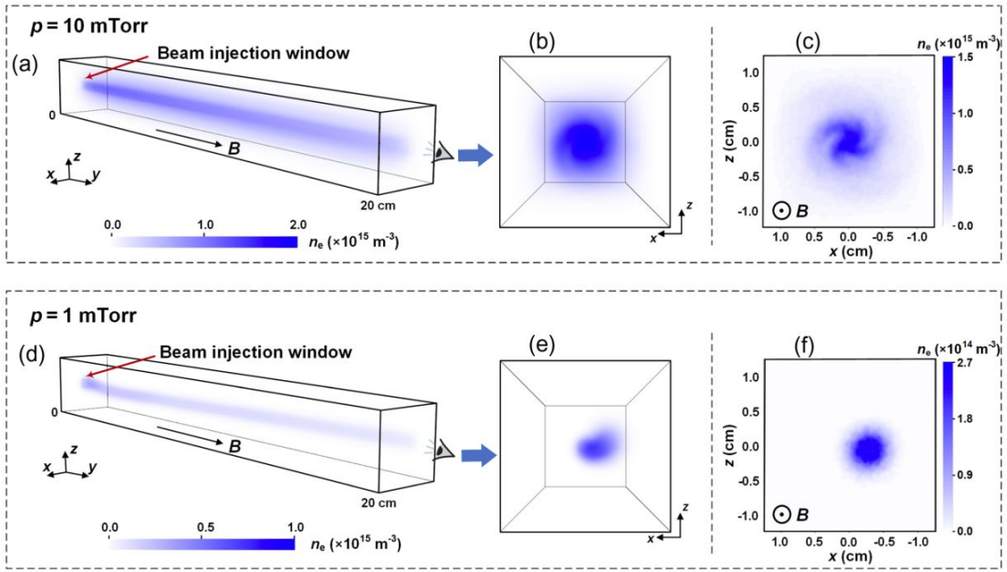
T Charoy et al. Plasma Sources Sci. Technol. 28 105010 (2019)



Instabilities in penning discharges

- Wide range of operation parameters
  - Coherent structures and oscillations.
  - Evidence of anomalous transport, especially at low pressures
  - Appearance of rotating structures.

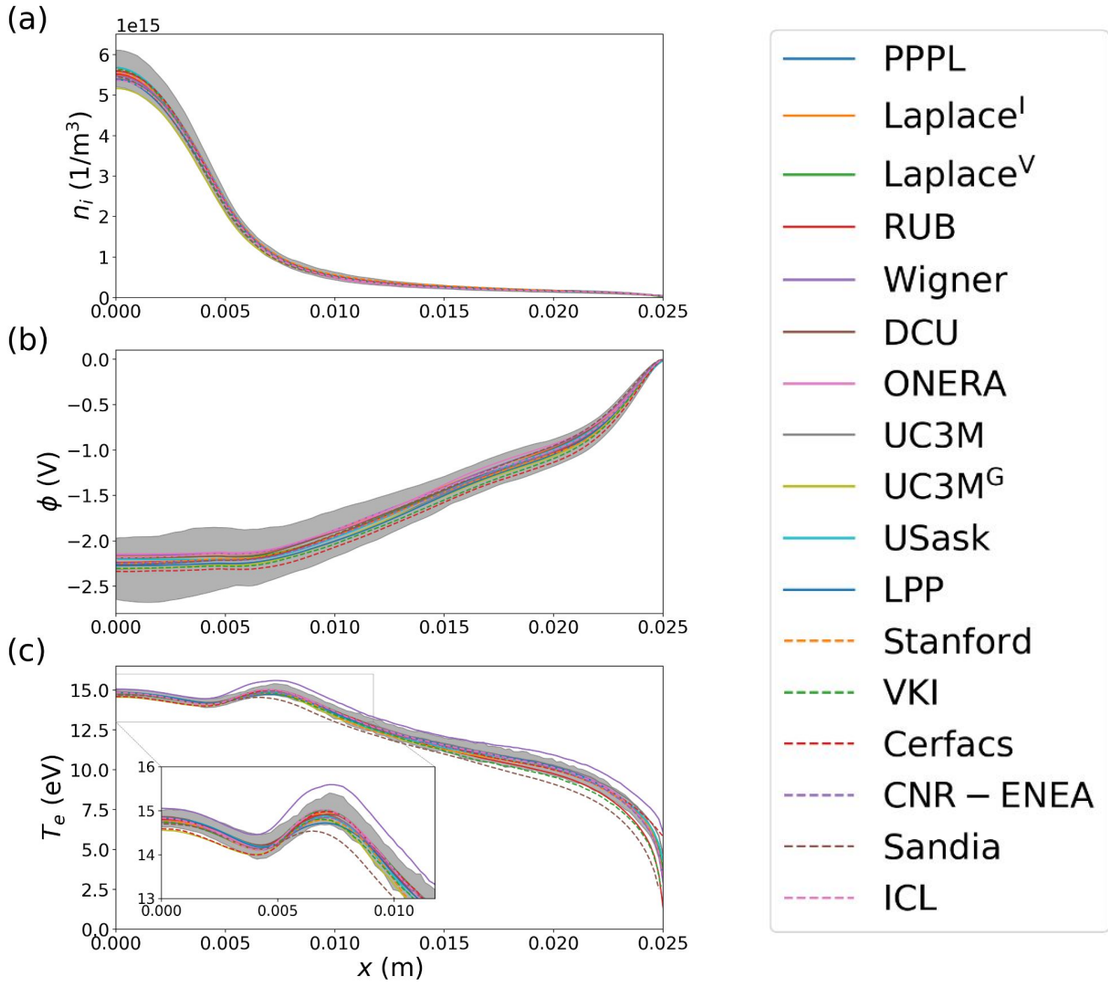
- There has been a recent interest in simulating Penning discharges by the **ExB** community

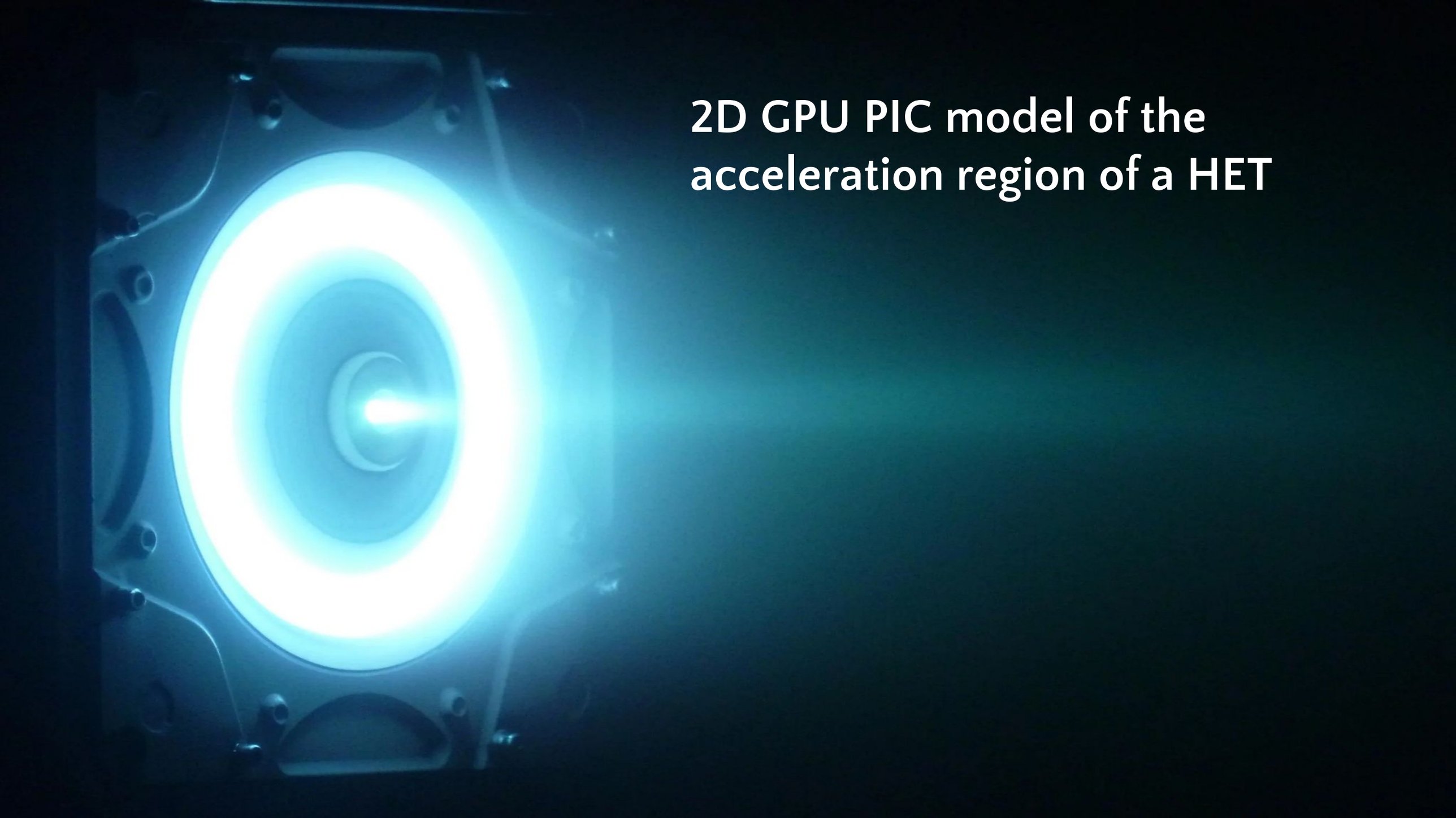


J Chen et al. Phys. Rev. Lett. 135 045301 (2025)

PIC code benchmark based on a Penning discharge plasma

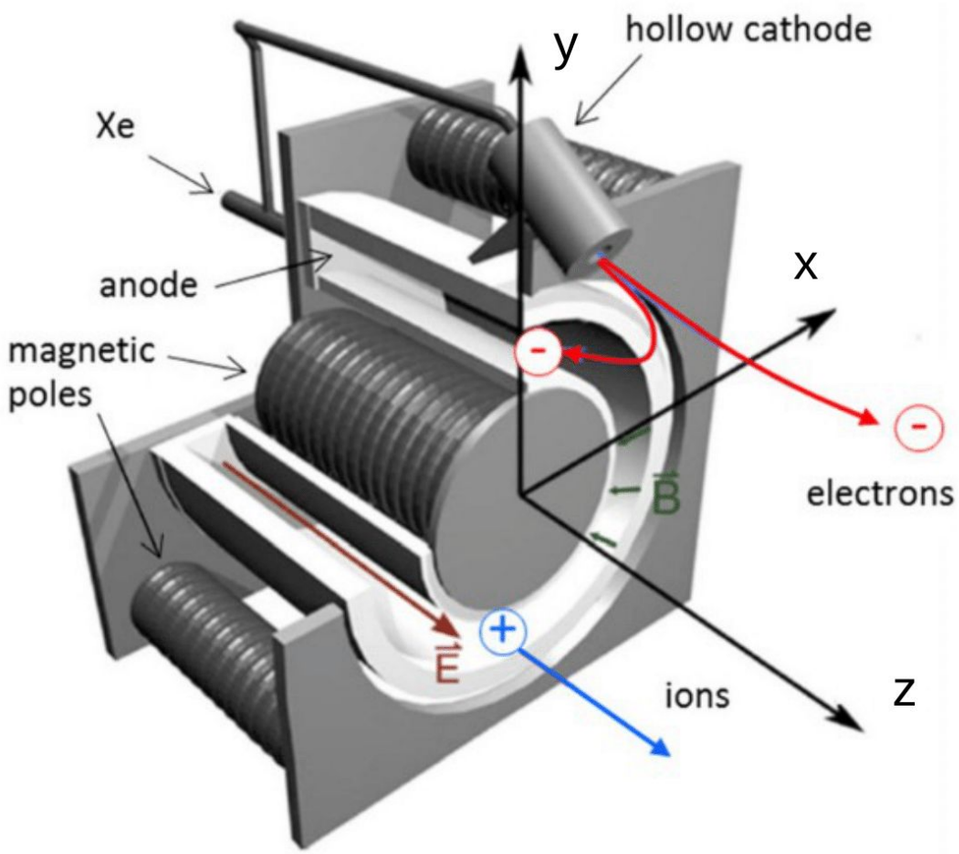
A. T. Powis et al. Plasma Sources Sci. Technol. 35 025002 (2026)




2D GPU PIC model of the
acceleration region of a HET

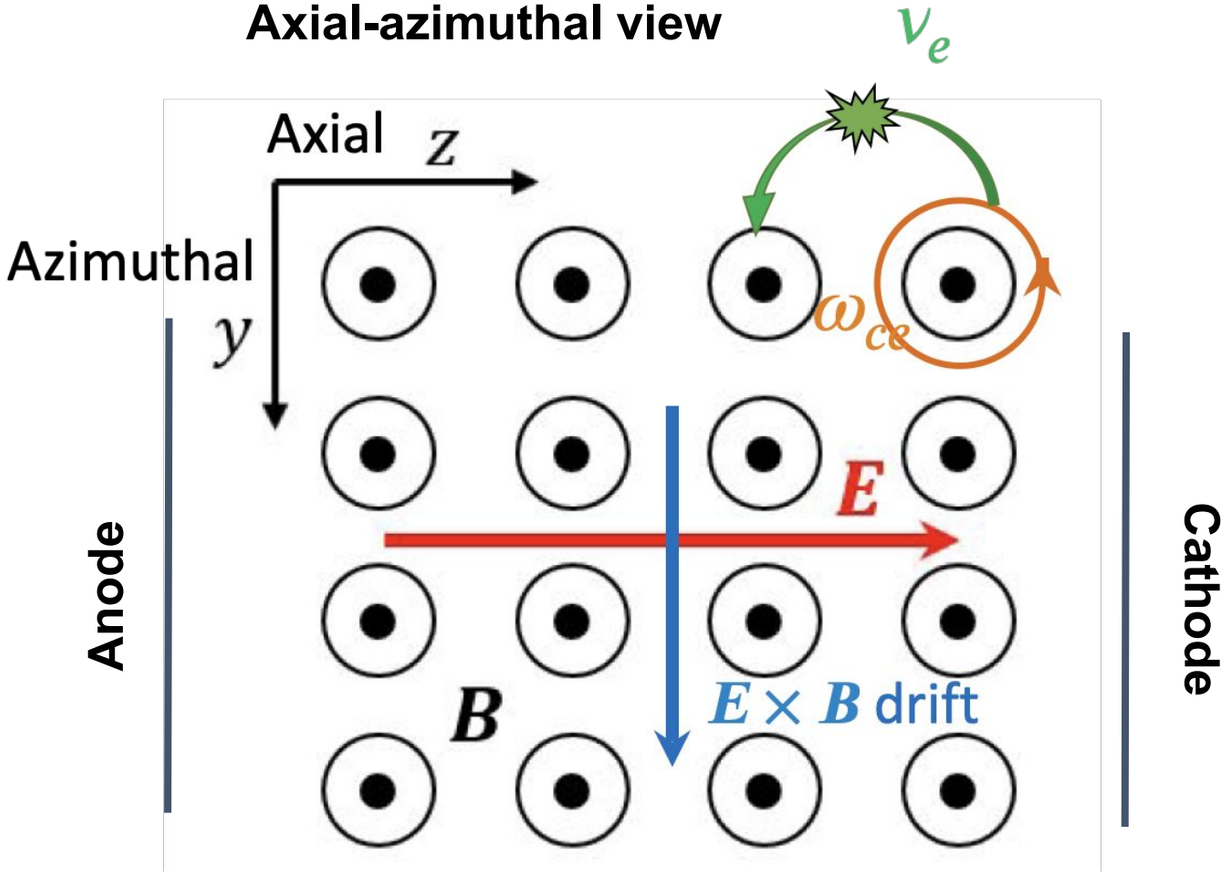
Anomalous electron transport in HET

HET sketch



J. Boeuf. J. Appl. Phys. 121, 011101 (2017)

Axial-azimuthal view



Axial electron transport possibly enhanced by
Azimuthal oscillations and plasma instabilities

Open problems

Identify and **characterize** the main **kinetic instabilities producing** the **collisionless anomalous electron transport**

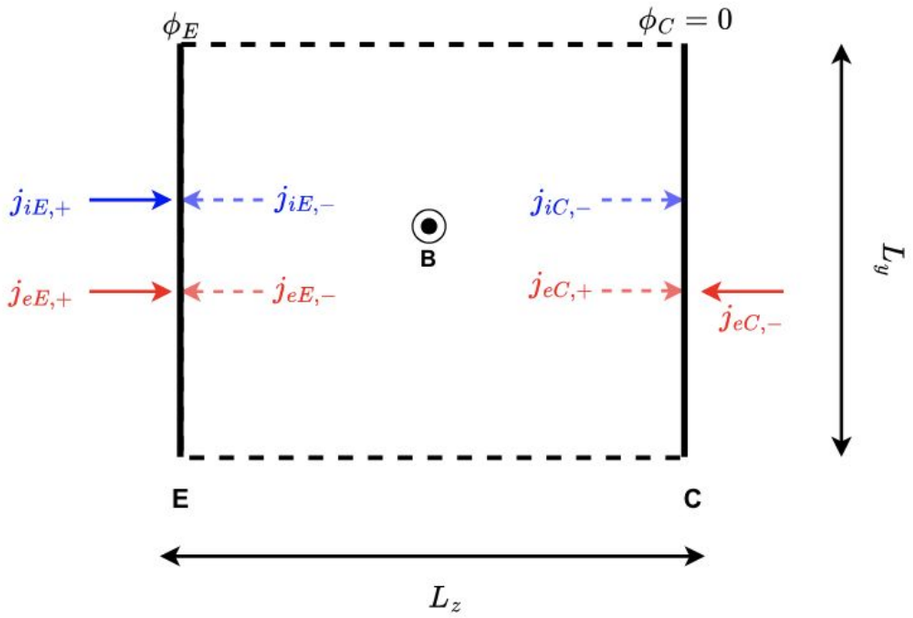
Quantify the **transport** for different plasma configurations

Collisionless particle in cell (PIC) simulations

GPU kinetic modelling

1. **Allows for massively parallelization → up to 10x - 100x faster than CPU-based codes**
2. **Improves statistical accuracy by simulating more particles**
3. **No artificial scaling on the vacuum permittivity**
4. **Allows high resolution + long-time simulations**
5. **Support fast parametric studies**

Simulation set up and model

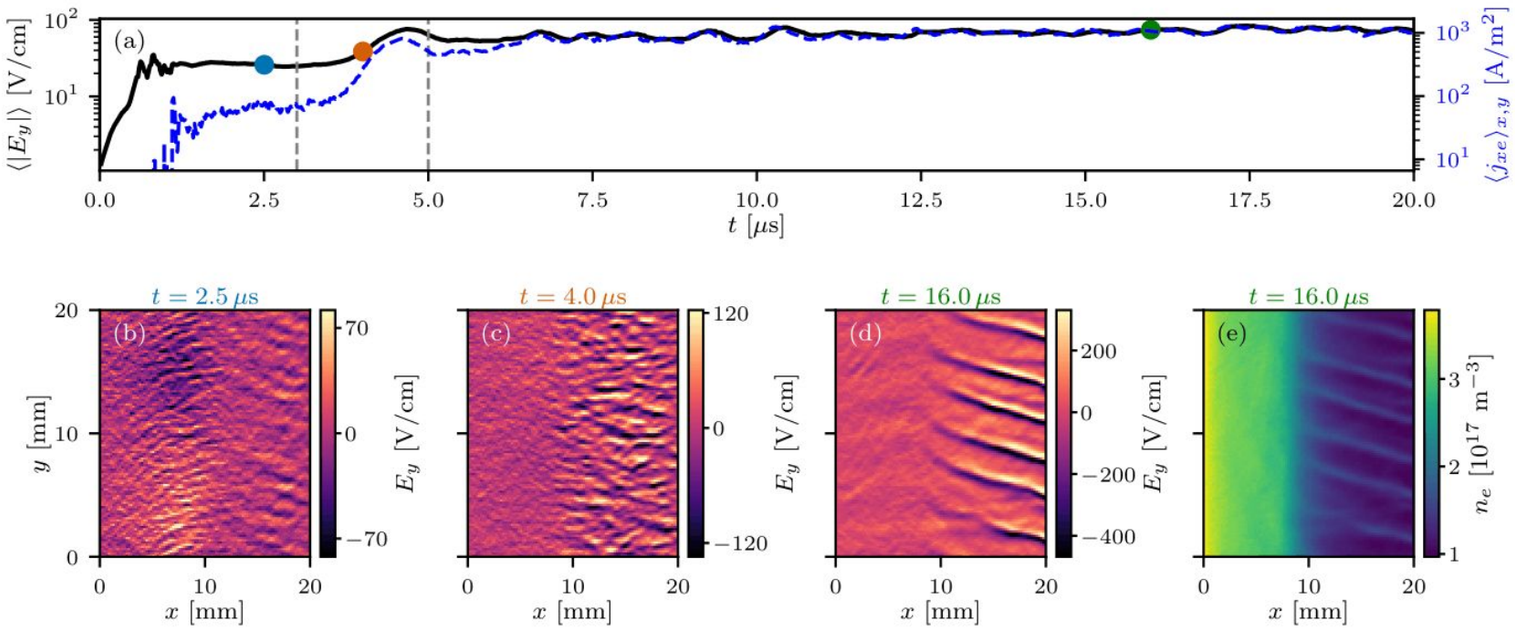


Simplified model of the **acceleration region** of a Hall Effect Thruster → **instabilities and anomalous transport are dominant**

- **Constant radial magnetic field, B = 50 G**
- **200 V Potential Drop:** Applied between E and C to mirror physical acceleration.
- No **direct electrical connection** between E and C.
- **Constant ion current injected at E**, based from plasma properties after ionization.
- **Constant electron injection at E and C to maintain quasineutrality**

Simplified configuration with the minimum number of free parameters which allows to **analyze the impact of instabilities without the added complexity of magnetic field gradients, collisions**, etc...

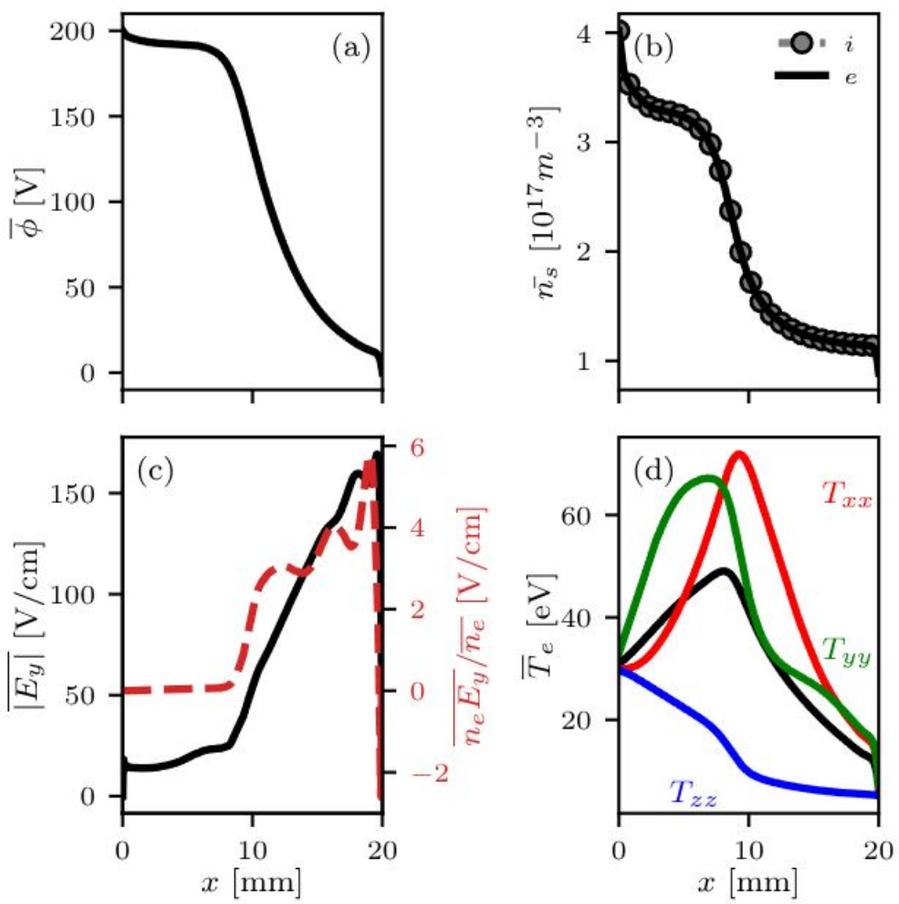
Onset, growth and saturation of instabilities



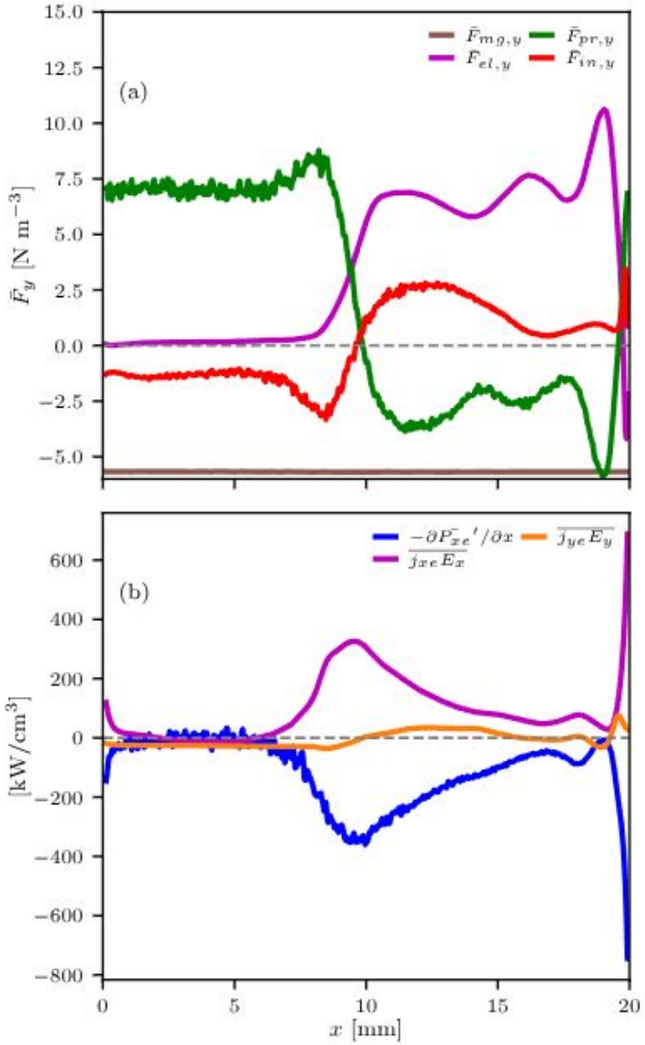
Plasma starts in an empty domain and self-organizes into a saturated turbulent state sustained entirely by internal field dynamics.

1. **Stage 1 (0 < t < 3 μs): Establishment** of a **quasi neutral plasma** form an empty domain.
2. **Stage 2 (3 < t < 5 μs): Exponential growth** of |Ey| and cross-field electron current; azimuthal structures
3. **Stage 3 (t > 5 μs): Persistent azimuthal structures**; fields and currents fluctuate around steady mean values.

1D saturated axial macroscopic profiles



Electron azimuthal momentum and energy balance



Quasineutrality & Ion Acceleration: Monotonic axial decay of electric potential from electrode E to C accelerates ions towards C

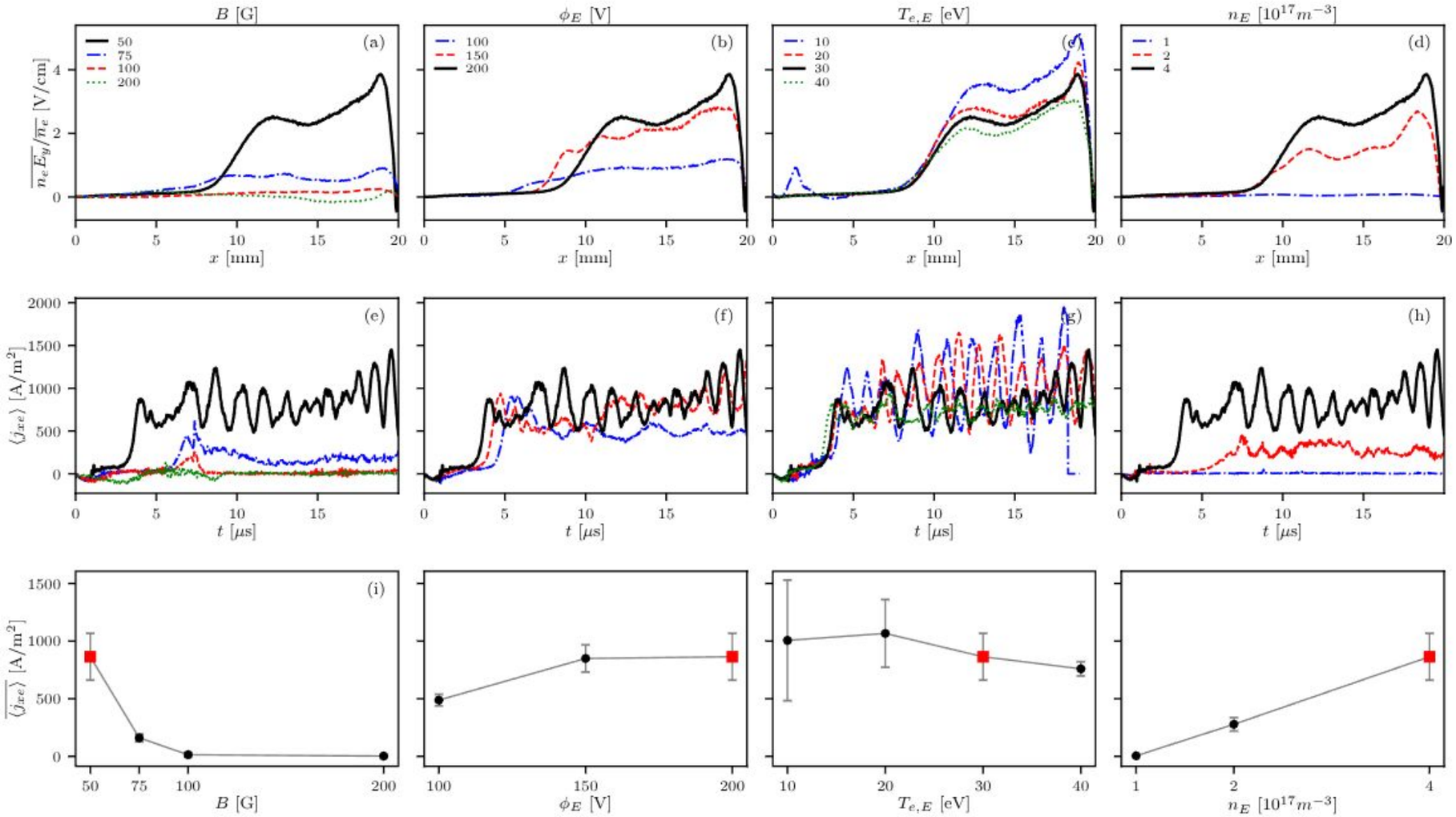
Azimuthal Structures & Transport: Fluctuation amplitude |Ey| increases axially; its correlation with electron density drives anomalous cross-field electron current peaks for x > 10 mm.

Anisotropic electron Heating: These azimuthal structures generate strong **anisotropic heating of electrons**, reflected in the electron temperature profiles.

Momentum Balance: Magnetic force is primarily balanced by **correlated oscillations** in the electron density and the azimuthal **electric field** (x > 10 mm) and **Finite Larmor Radius (FLR) effects**, (x < 10 mm)

Energy Balance: Energy flux balance by the power of the **axial electric field** with **small contribution** from the power of the **azimuthal electric field.**

Parametric studies



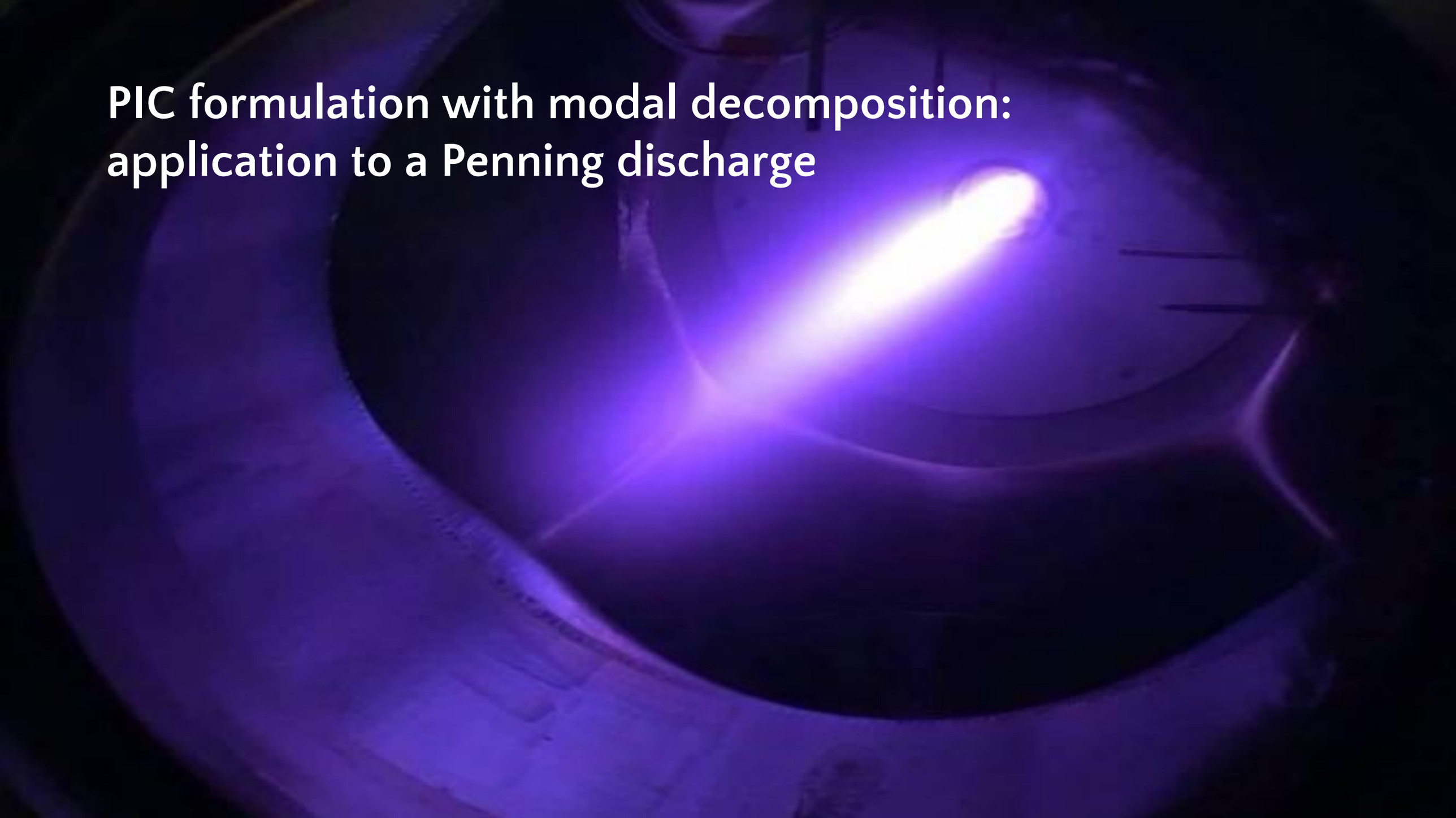
Magnetic Field: **High field strength quenches anomalous cross-field current** (role of the gradient of the magnetic field remains open).

Bias Voltage: **Alters ExB drift** and instability growth, but response saturates above 150 V.

Injection Temperature: **Modulates current oscillations via FLR effects,** leaving the instability region and mean current mostly unchanged.

Electron Density: **Cross-field electron current scales linearly** with injected plasma density.

PIC formulation with modal decomposition: application to a Penning discharge



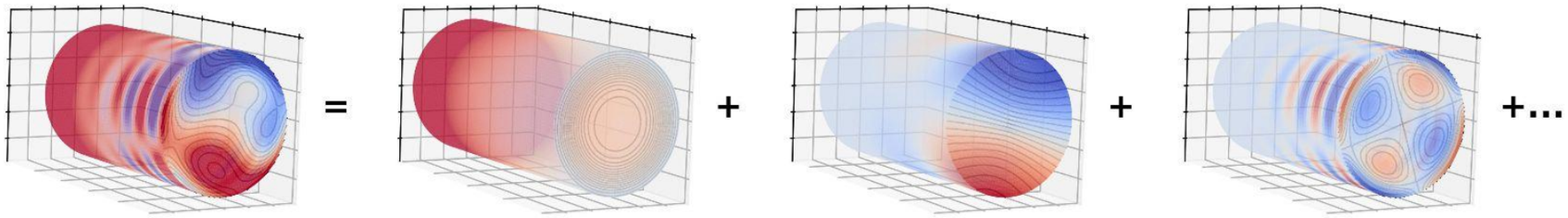
1D/2D+1F PIC code

- Finite difference (FD) PIC
  - Particles weighted on a grid
  - Fields interpolated from grid → aliasing

- Most ExB discharges are cylindrical
  - Fields are periodic in θ → **Fourier** decomposition!
  - Particles can be weighted spectrally in θ
    - and on a grid in r in 2D, and to (z,r) in 3D
  - Improved spectral fidelity (no aliasing in 1 direction)
  - Problem of D dimensions → N+1 problems of (D–1) dimensions

$$n(t;z,r,\theta) = n^0(z,r) + \sum_m^{N_m} \Re\left[n^m(t;z,r)e^{im\theta}\right]$$

$\psi(z,r,\theta)$ $\psi_0(z,r)$ $\psi_1(z,r)\, e^{i\,\theta}$ $\psi_2(z,r)\, e^{i\,2\theta}$



1D+1F PIC algorithm

- 2D example: FD in r + Fourier in θ
  - Physical grid only in r
  - Gridless azimuthally
- For N_m spectral modes:
  - Particles are weighted on the r-grid
  - Particles are weighted on each θ-mode
- 2D problem becomes (N_m+1) Poisson problems
  - Each mode is solved independently
- Electric field interpolated in r and evaluated in θ
  - Modes are coupled back together
  - The direct evaluation is what eliminates aliasing in θ!
- Particles are pushed following Boris' algorithm

$$n_e(r,\theta,t) = n_e^0(r,t) + \Re\left[\sum_{m=1}^{N_m} n_e^m(r,t)\exp(im\theta)\right]$$

$$n_e^0(r_i) = \sum_p \frac{w_p}{2\pi r_i \Delta r} S_r(r_p, r_i),$$

$$n_e^m(r_i) = \sum_p \frac{w_p}{\pi r_i \Delta r} S_r(r_p, r_i)\exp(-im\theta_p) \quad \text{for } m > 0\,,$$

$$\nabla^2\phi^m \equiv \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi^m}{\partial r}\right) - \frac{m^2}{r^2}\phi^m = -\frac{\rho^m}{\varepsilon_0}$$

$$\boldsymbol{E}(r,\theta) = \boldsymbol{E}^0(r) + \Re\left[\sum_{m=1}^{N_m} \boldsymbol{E}^m(r)\exp(im\theta)\right].$$

Verification of 1D+1F against planar case: Penning Discharge

- Cylindrical Penning discharge
  - 2D case: reduction to 1D+1F
  - Non-neutral injection
    - Radial Heaviside injection profile
    - Particles are injected at random r and θ inside injection region

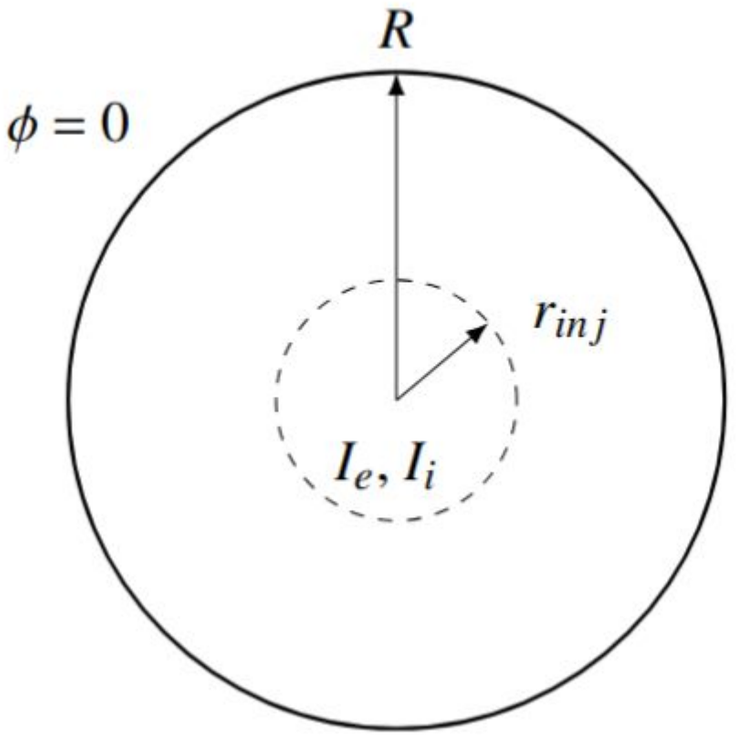
$$r_p = r_{inj}\sqrt{x_1}, \qquad \theta_p = 2\pi x_2.$$

  - Different BCs at each mode
    - at r=0:

$$\left.\frac{\partial \phi^0}{\partial r}\right|_{r=0} = 0. \qquad \phi^m|_{r=R} = 0.$$

    - at r=R:

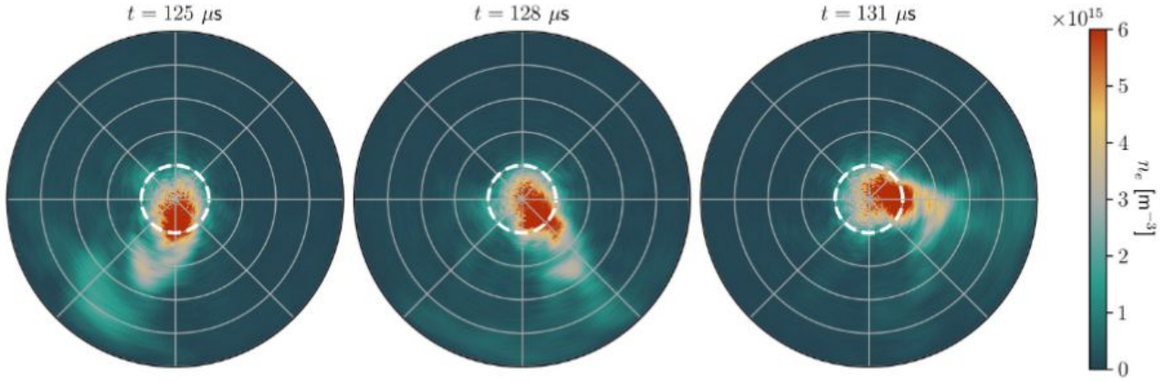
$$\phi^m|_{r=0} = 0 \quad \text{for } m > 0.$$

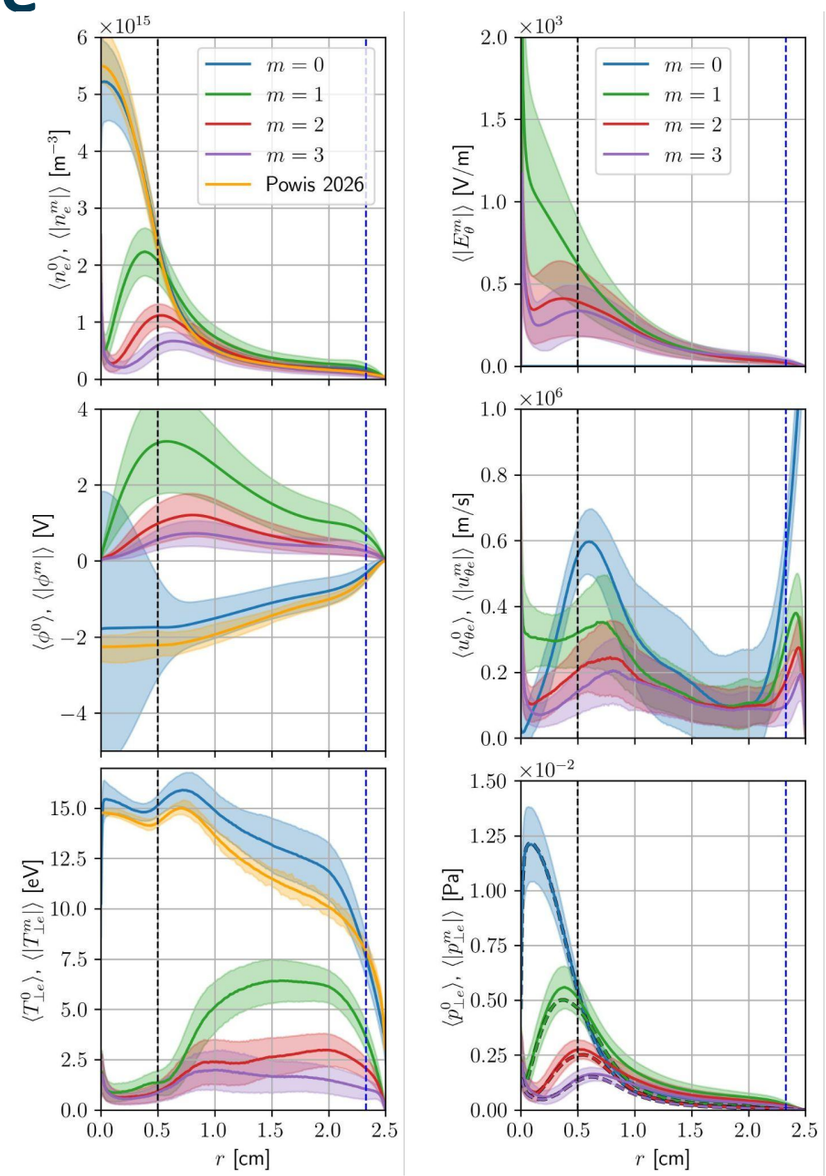


Results: reference case

- Reference: N_m=20
  - Quasineutral discharge
  - Dominance of m=1
    - For all fields
  - Comparison with benchmark simulation (for m=0)

    A. T. Powis et al. Plasma Sources Sci. Technol. 35 025002 (2026)
    - Reasonable agreement in ne, Φ, T_e
    - Differences in geometry, particle shape…
  - Comparing p_e^m with T_e^m x n_e^m
    - Validity of isothermal wave assumption (usual in SHI models)





Results: reference case

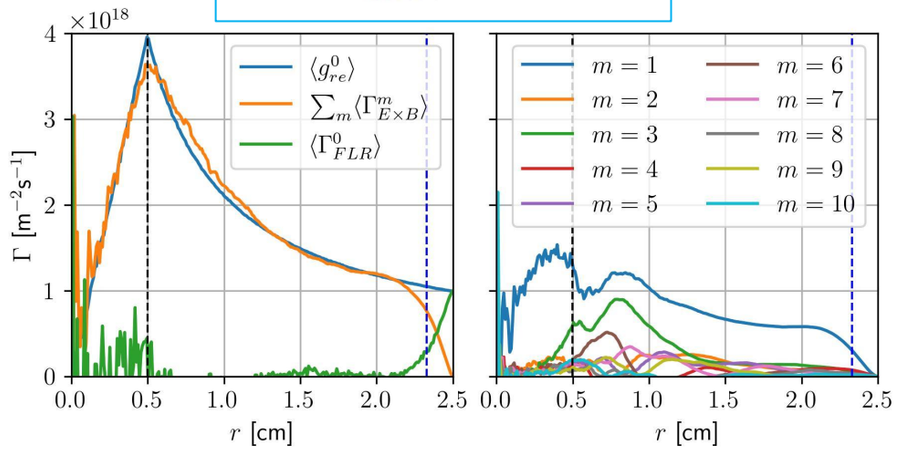
- Reference: N_m=20
  - With Fourier decomposition, contribution of each mode to ExB cross-field transport can be studied
    - m=1 and m=3 contribute the most
    - Contribution of other modes is nonzero
    - FLR effects at Larmor diameter from the wall
  - Direct implementation of spectral diagnostics
    - PSD of ne reveals frequency of m=1 at 48 kHz (43.35 in benchmark!)
    - Rotating spoke has phase velocity of sonic wave at the wall r=R
    - Coherence between ne and Φ shows strong correlation at m<3

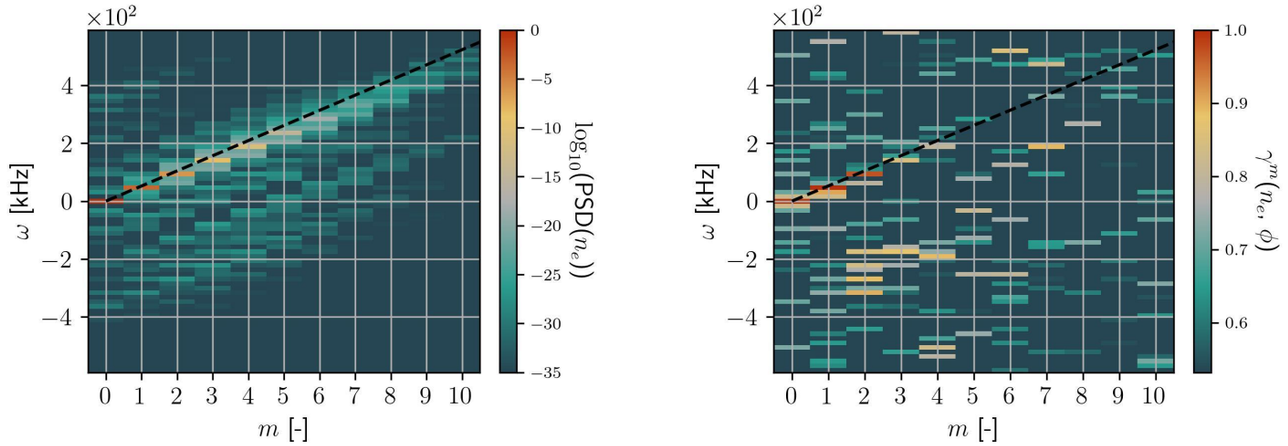
wave-driven transport

electron radial m=0 flux

gyroviscous contribution

$$\langle g^0_{re} \rangle = \sum_{m=1}^{N_m} \Gamma^m_{E\times B} + \Gamma^0_{FLR},$$



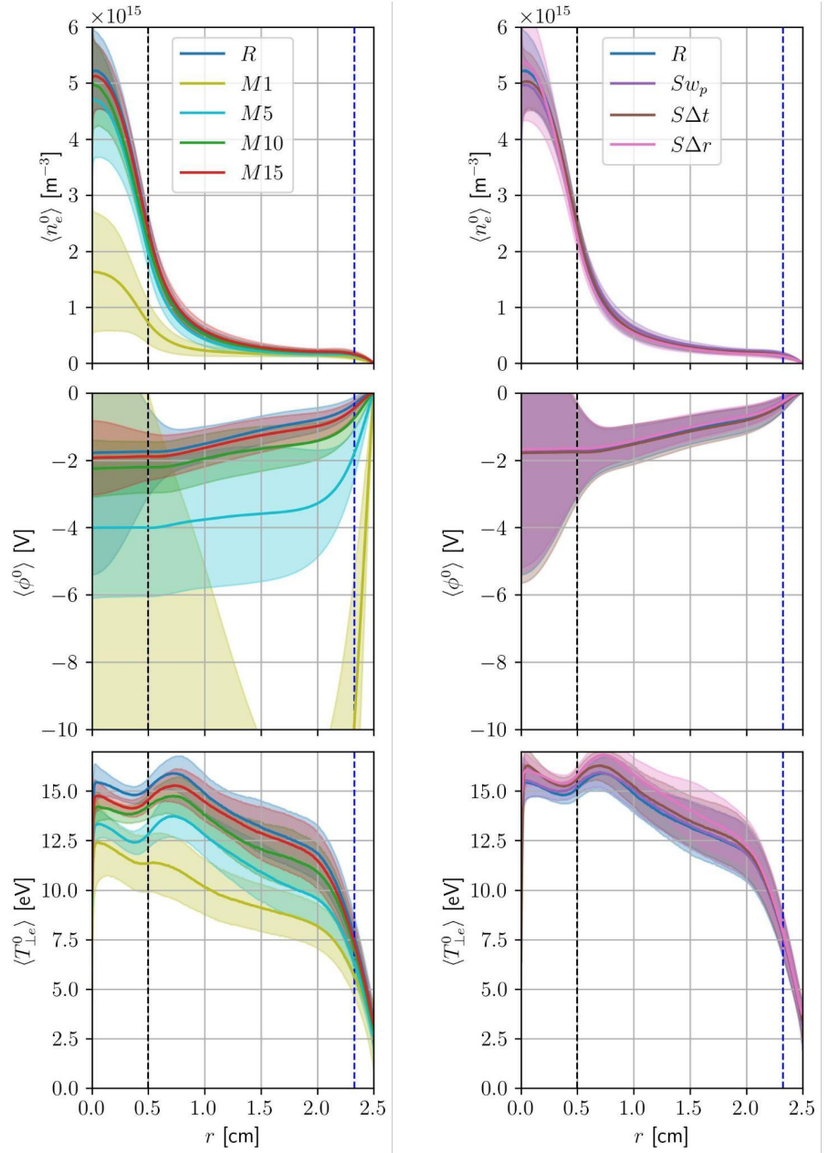


Convergence study and computational costs

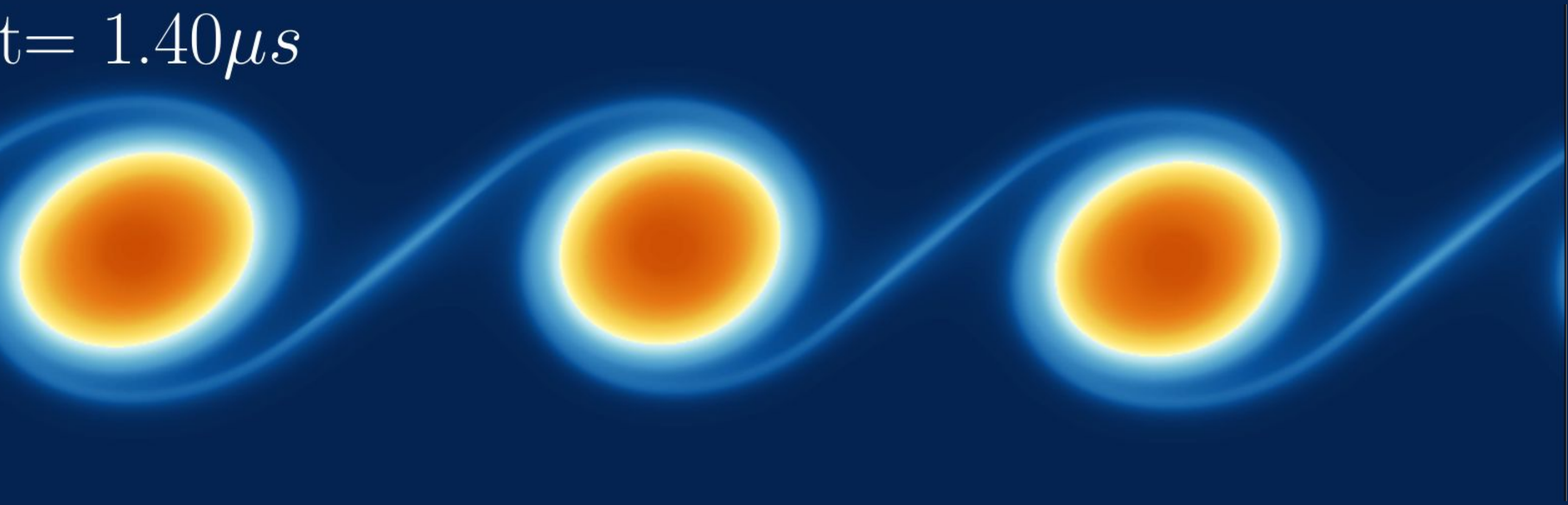
- Convergence in N_m, Δt, Δr, w_p
  - Convergence for $N_m > 10$
  - Relaxing spatio-temporal grid does not damage results
  - Heavier macroparticles lead to same steady-state
- Computational cost:
  - Simulations ran on 16 physical cores
  - Weak dependence on N_m, strong on Np
  - Compare with PICASO (2D, FD)
    - 16 cores of 1D+1F vs 40 cores of 2D FD
    - twice the number of timesteps to reach 500 μs
    - → speedup of a factor of ~8

NOTE: spatio-temporal grid 2x finer then benchmark

Case	N_m	w_p [10^6]	Δt [10^{-2} ns]	Δr [10^{-2} cm]	Wall time [h]	Total particles [10^6]
R	20	2	2	1	110	1.8
M1	**1**	2	2	1	81	1.2
M5	**5**	2	2	1	95	1.6
M10	**10**	2	2	1	103	1.7
M15	**15**	2	2	1	108	1.8
Sw_p	20	**4**	2	1	59	0.8
SΔt	20	2	**4**	1	65	1.8
SΔr	20	2	2	**2**	114	1.9



Full-fluid modelling of ExB instabilities



5-moment fluid model (MEPHYSTO 5-M)

Main characteristics

- Scalar temperature
- No heatflux
- 2D-2V model

Solution method

- Finite volume with HLLC solver
- Fractional step method + SSP-RK3 for time integration

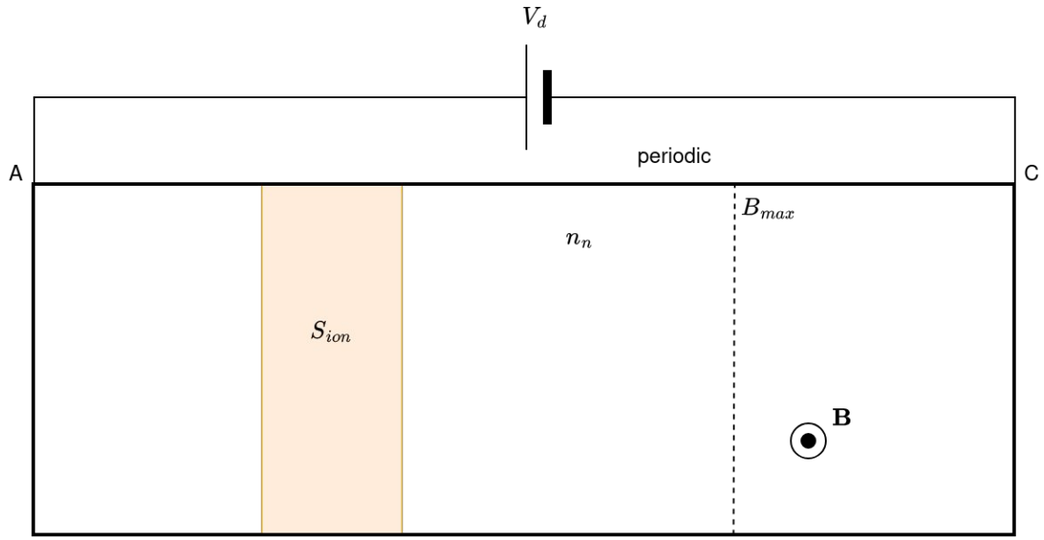
$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \boldsymbol{u}_\alpha) = S_\alpha$$

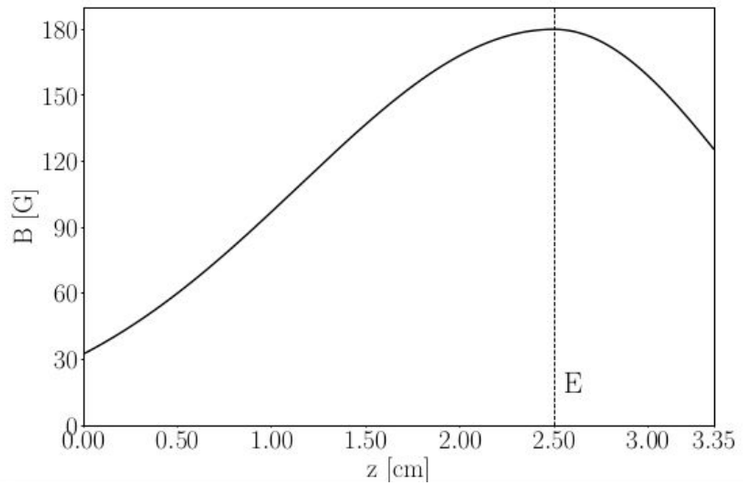
$$\frac{\partial (n_\alpha \boldsymbol{u}_\alpha)}{\partial t} + \nabla \cdot (n_\alpha \boldsymbol{u}_\alpha \boldsymbol{u}_\alpha) = \frac{q n_\alpha}{m_\alpha} [\boldsymbol{u}_\alpha \times \boldsymbol{B} - \nabla \phi] - \frac{1}{m_\alpha} \nabla p_\alpha + \boldsymbol{R}_\alpha$$

$$\frac{\partial (\mathcal{E}_\alpha)}{\partial t} + \nabla \cdot ((\mathcal{E}_\alpha + p_\alpha) \boldsymbol{u}_\alpha) = q n_\alpha \boldsymbol{u}_\alpha \cdot \nabla \phi + P_\alpha$$

$$\mathcal{E}_\alpha = \tfrac{3}{2} n_\alpha T_\alpha + \tfrac{1}{2} m_\alpha n_\alpha u_\alpha^2$$

HET-like simulation





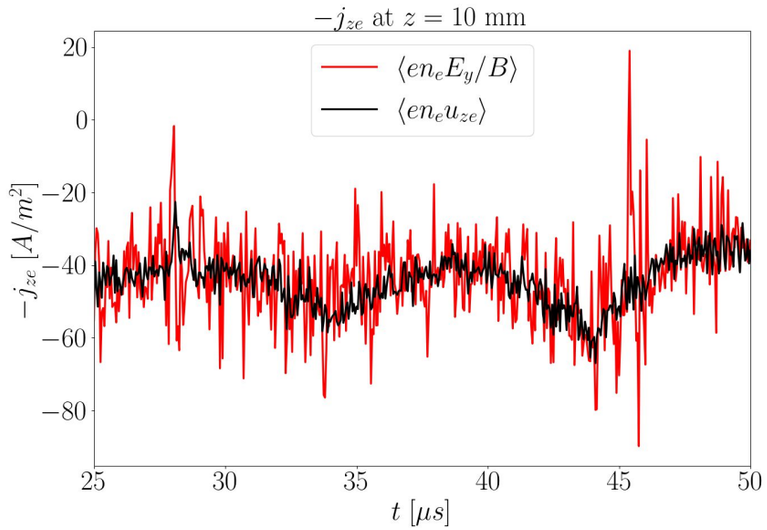
Full-fluid z-theta HET simulations (Poli et al. 341 – IEPC25):

- Superposition of
  - Rotating spokes
  - Breathing mode
  - Short-wavelength instability
  - Ion transit time instability
- Complex and long simulation

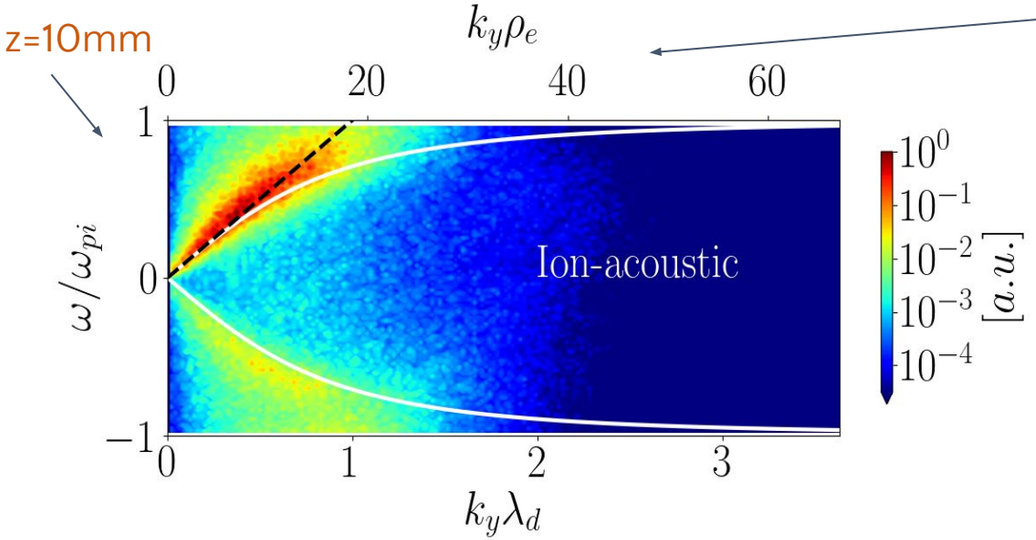
Simplified axial-azimuthal HET geometry

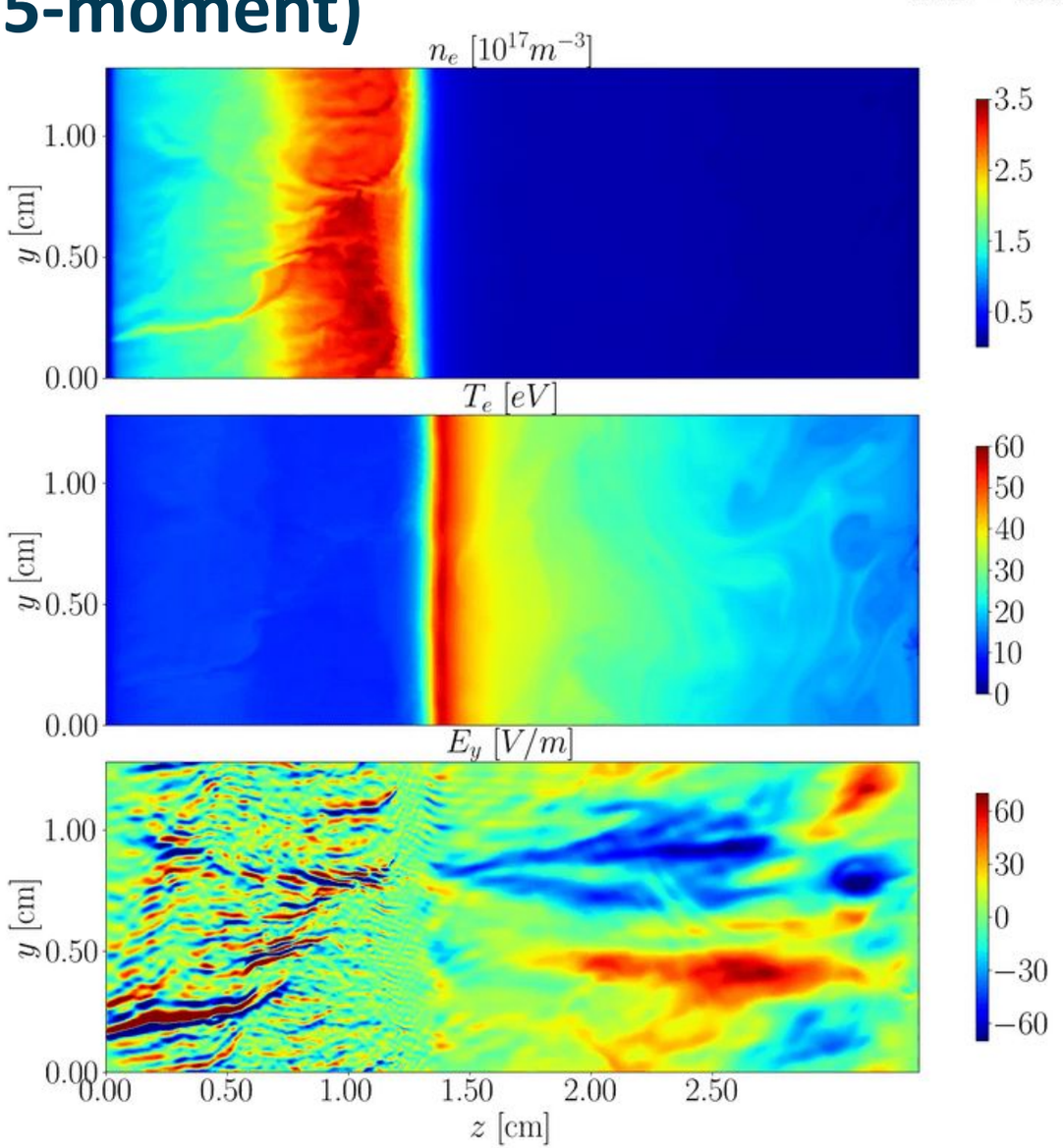
- Fixed ionisation source
  - No neutrals dynamics
- Uniform neutral background for elastic collisions
- Representative magnetic field
- 50 μs, 1.28x3.35 cm domain
- Faster to simulate (than IEPC simulation)

HET-like simulation (5-moment)



Anomalous current is dominant





10-moment fluid model (MEPHYSTO 10-M)

Main characteristics

- Full pressure tensor (6 energy equations)
- No heatflux
- Collisional terms from BGK model [1]
- Collisional rates assume a Maxwellian (scalar fluid temperature)

Solution method

- 2D-3V model
- Finite volume with HLLC solver
- Fractional step method + SSP-RK3 for time integration

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x_j}(n u_j) = S_p$$

$$m\frac{\partial n u_i}{\partial t} + \frac{\partial}{\partial x_j}\mathcal{E}_{ij} = nq(E_i + \epsilon_{ijk} u_j B_k) + R_c$$

$$m\frac{\partial \mathcal{E}_{ij}}{\partial t} + \frac{\partial}{\partial x_j}\mathcal{Q}_{ijk} = nq u_{[i} E_{j]} + \frac{q}{m}\epsilon_{[ikl}\mathcal{E}_{kj]} B_l + P_c + \delta_{ij} S_p T_{inj}$$

Where

$$\mathcal{E}_{ij} = P_{ij} + nm u_i u_j, \quad \mathcal{Q}_{ijk} = Q_{ijk} + u_{[i}\mathcal{E}_{jk]} - 2mn u_i u_j u_k \quad \text{with} \quad Q_{ijk} = 0$$

[1] J.M.Burgers-Flow Equations for Composite Gases, 1969

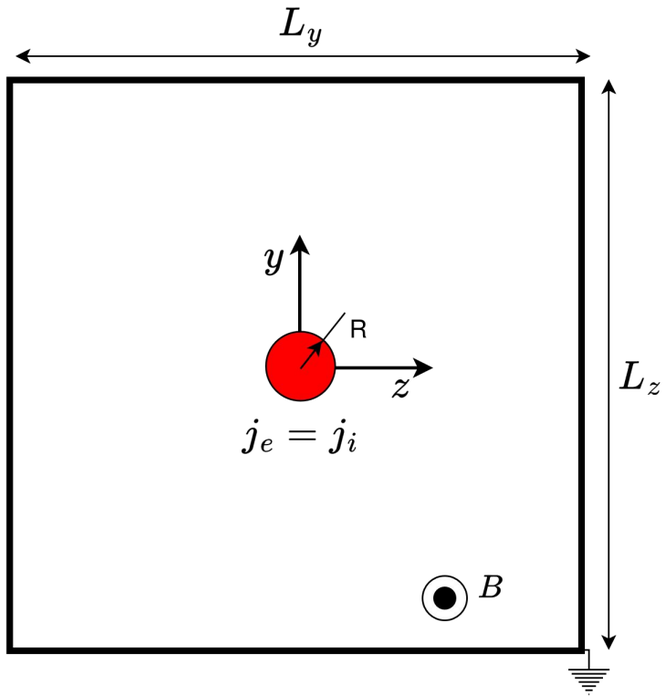
Magnetised Plasma column test

Plasma column to test the capability of MEPHYSTO 10-M:

- To simulate 2D ExB plasma scenarios
- To capture plasma instabilities
  - While being robust enough to not explode

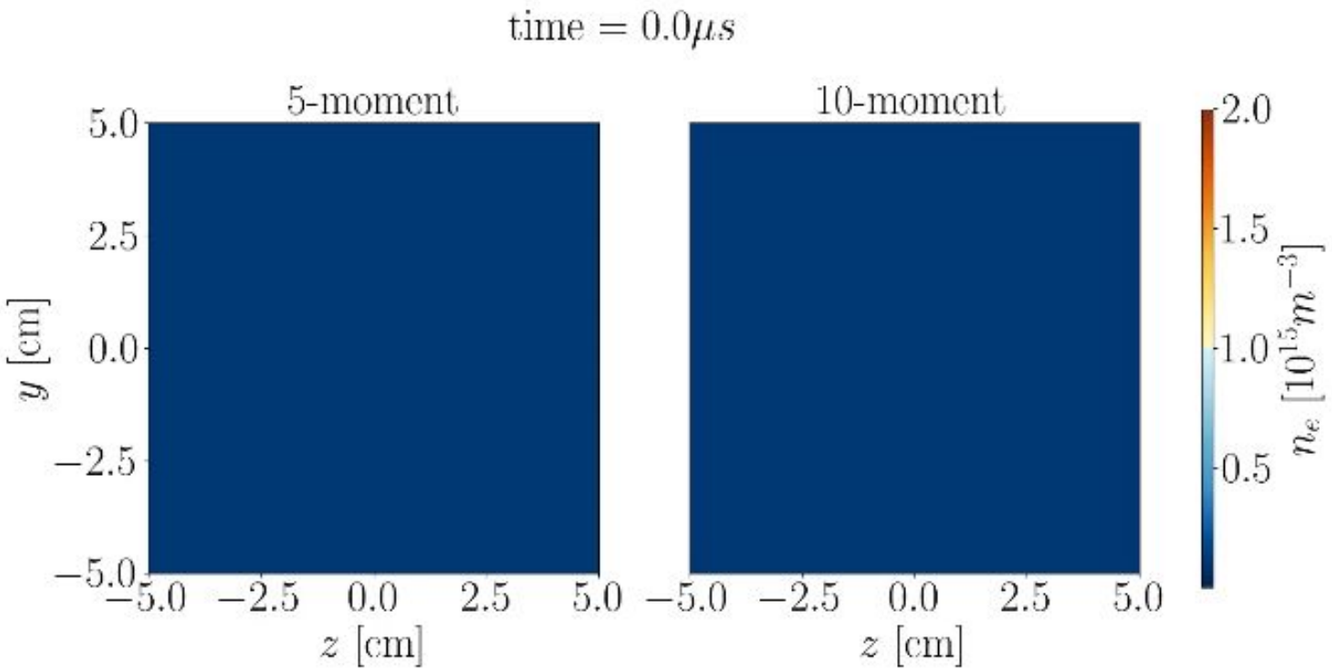
Standard magnetised plasma column scenario:

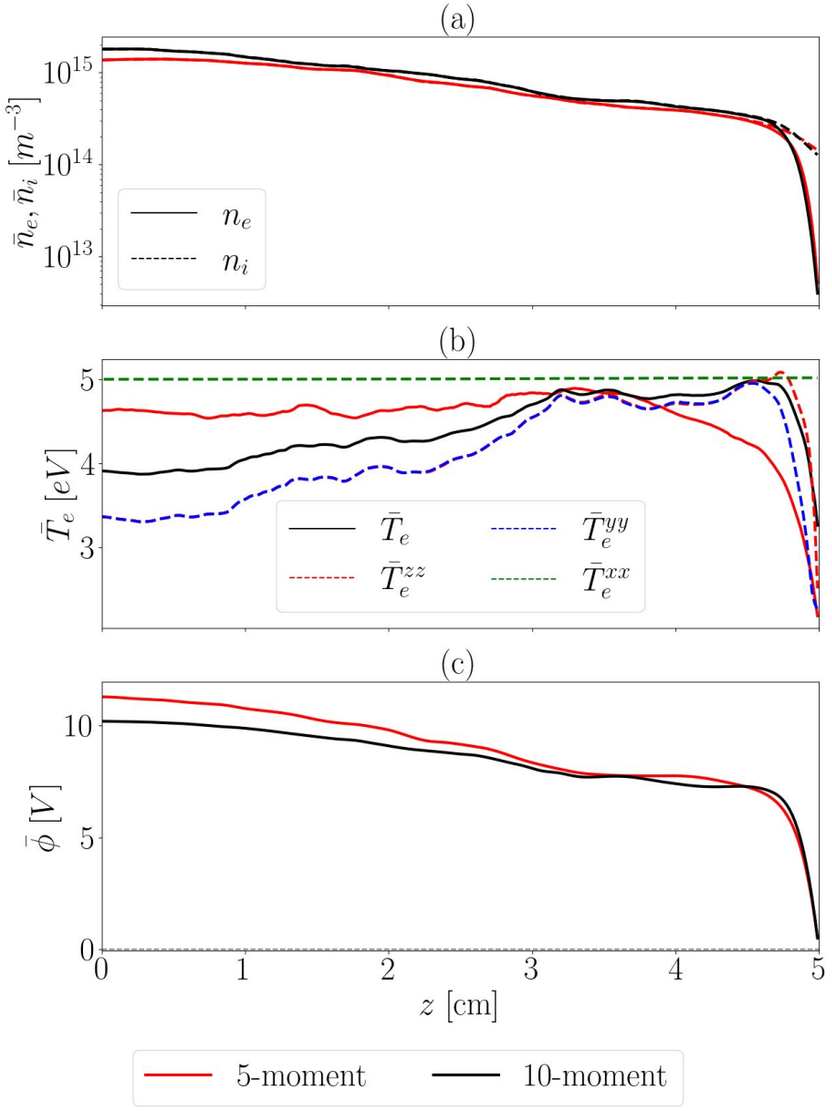
- Quasi-neutral injection
- Grounded walls
- Argon, collisionless
- B=100G
- 10x10 cm, 512x512 cells
- Simulated time 50 μs



Magnetised Plasma column test

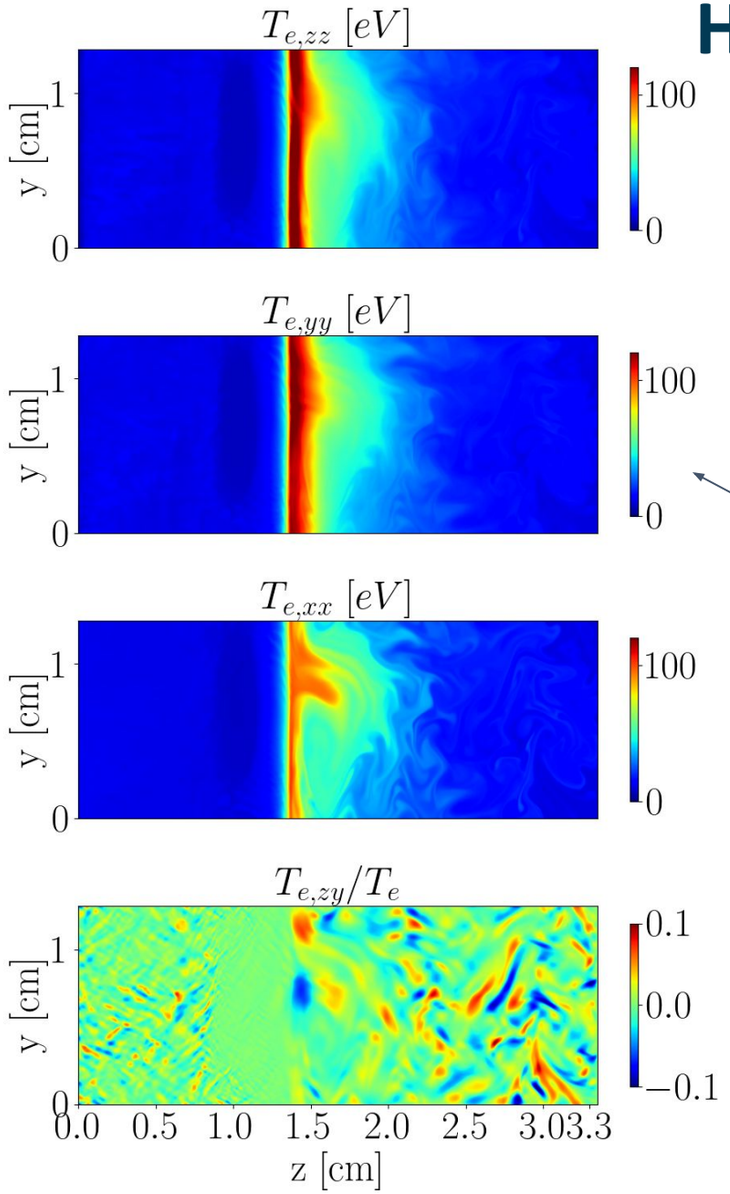
Comparison with 5-M





HET-like simulation (10-moment)

Very preliminary results

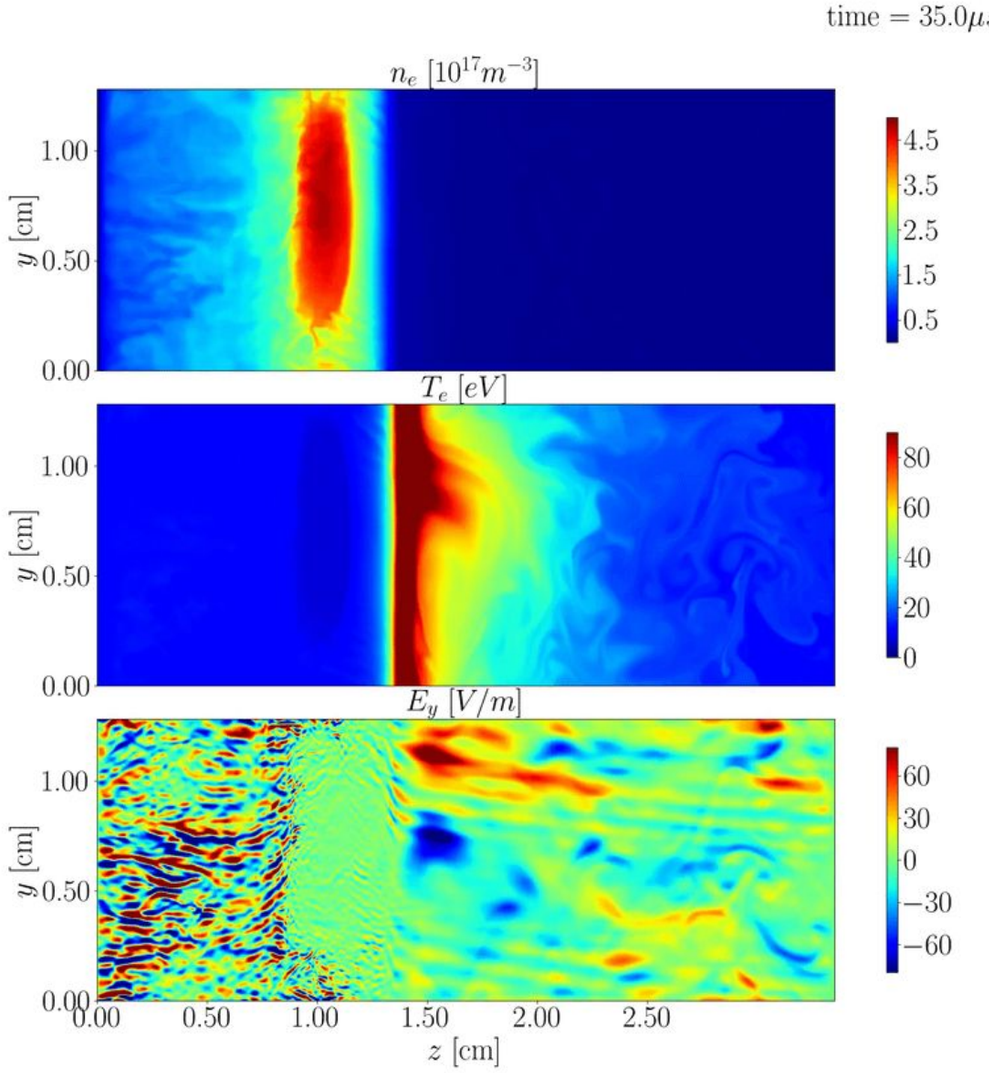


Longer wavelength?

Instantaneous pressure tensor (Te,xy=Te,zx=0)

To be compared with:

- Mephysto 5-m
- PICASO (PIC)



Conclusions and future work

- Anomalous transport is a long standing problem in ExB discharges
  - Understanding and characterizing anomalous transport is crucial to develop predictive models of the plasma discharge.
  - We approach this problem from a computational perspective, using particle-in-cell and fluid models with application to Hall Effect Thrusters (HETs) and Penning discharges.
- A GPU PIC model to analyze the acceleration region of a HET
  - A simplified test case is designed to analyze the role of plasma oscillations in electron transport.
  - Allows to perform computationally challenging simulations in a reasonable time.
  - Parametric studies
    - Numerical parameters: ensure the robustness of simulation results
    - Physical parameters: extract conclusions on the effect of certain quantities on anomalous transport.
- A PIC formulation with modal decomposition
  - Promising technique showing good accuracy and a competitive runtime in 2D (1D+1F)
  - Application to 3D (2D+1F) is the next step
    - Benchmarking and cross verification with 3D codes
    - Simulation of real devices
- Fluid models of ExB discharges
  - Fluid models capture the wave induced anomalous transport in ExB discharges.
  - A 10-moment model is developed to include gyroviscosity and anisotropy (full pressure tensor)
  - The effect of the heat flux and higher order moments is still an open question
- The combined work with kinetic and fluid formulations can help to unveil the origin of anomalous transport.

Thank you! Questions?

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email: ep2@uc3m.es
web: ep2.uc3m.es

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EXCELENCIA MARÍA DE MAEZTU

Summary and Future works

- PIC+Fourier seems promising
  - Competitive runtime
  - Easy to retrieve mode dynamics separately

- Unoptimized code
  - Smoother particle shapes may help reduce noise
  - Port to GPU seems natural step

- Next: application to 3D
  - Possible big memory saving (no need for 3D grid)
  - Comparison with 3D cartesian FD code for cross-verification
  - Application to real thruster plumes
    - "cheap", self-consistent turbulence analysis
    - can capture 3D sources and skew symmetries in the plume!
      - cathodes in HTs, GITs
      - magnetic uniformities from HT magnetic circuit
      - non-axisymmetric power deposition in ECRs

Conclusions and future works

- Fluid models can help investigate the origin of the anomalous transport (with PIC)
  - Mephysto 5-m simulations capture the instability induced transport
  - The resulting wave (close to ion-acoustic) is outside of the limit of validity of 5-m models

Mephysto 10-m model includes gyroviscous effects (full pressure tensor) to capture (qualitatively) $k\rho \gg 1$

- Preliminary simulations with Mephysto 10-m
  - Successful 2D simulations of ExB plasmas
  - Capability to resolve rotating structures in a plasma column
  - Capability to perform a HET like simulation

- Future works
  - Validation of Mephysto-10m
  - Cross comparison between 5-m, 10-m and PIC simulations of HET geometry