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Fast Track Communication

Plasma detachment in a propulsive magnetic nozzle via ion demagnetization

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Abstract
Plasma detachment in propulsive magnetic nozzles is shown to be a robust phenomenon caused by the inability of the internal electric fields to bend most of the supersonic ions along the magnetic streamtubes. As a result, the plasma momentum is effectively ejected to produce thrust, and only a marginal fraction of the beam mass flows back. Detachment takes place even if quasineutrality holds everywhere and electrons are fully magnetized, and is intimately linked to the formation of local electric currents. The divergence angle of the 95%-mass flow tube is used as a quantitative detachment performance figure.

Keywords: electric propulsion, magnetic nozzles, plasma detachment, magnetohydrodynamics

(Some figures may appear in colour only in the online journal)

Magnetically channeled plasmas are fundamental in many applications, ranging from material treatment to fusion and space propulsion. In particular, several plasma thruster concepts [1–6] employ, as their acceleration stage, a longitudinal magnetic field to create a divergent magnetic nozzle (MN), which transforms the internal energy of the plasma into axially directed kinetic energy, thus producing a highly supersonic plasma beam [7]. While the purpose of a MN is the same as in a solid, de Laval nozzle [8], there are several differences in their operation: firstly, a MN has the key advantage of working in a contactless manner, thereby mitigating plasma losses and wall durability problems. Secondly, the expansion physics are more complex, far from one-dimensional [9], and depend strongly on the form in which the internal plasma energy is stored, which is device-dependent [10, 11]. Finally, there are significant differences in the processes of *thrust production* and *plasma detachment from the nozzle* due to the presence of the long-range electromagnetic forces.

The mission of any propulsive nozzle is to increase the delivered thrust. In a solid nozzle, the physical mechanism enabling this is the pressure exerted on the nozzle divergent walls. In a thruster with a wall-less MN, as we showed theoretically [9] and Takahashi *et al* later confirmed experimentally [12], thrust has two contributions: *pressure thrust* inside the plasma source and *magnetic thrust* from the MN: the magnetized plasma develops diamagnetic electron azimuthal currents that oppose those running in the thruster coils, thus creating two repelling magnetic forces: one yields positive thrust on the thruster, while the second one accelerates the plasma outward [13]. The MN can thus increase the total thrust of the initially sonic plasma jet by roughly a factor of two [9], and even a larger one if the plasma contains two disparate electron populations [14, 15]. The actual thrust gain depends on the thermodynamic behavior of the plasma and the MN strength and divergence losses.

There is ample consensus that the main uncertainty about the applicability of MNs in space propulsion is the ability of the plasma to detach efficiently from the closed magnetic lines [16]: once accelerated, it is imperative that the plasma beam releases itself from the magnetic field and continues to expand freely downstream. If this were not the case, the returning plasma would cancel the produced thrust and impinge on the spacecraft. Currently, experimental evidence of detachment is still incomplete, and its theoretical base is much debated. In one of the prevailing theories on the detachment of a globally current-free, divergent plasma jet, resistivity [17] or electron inertia [18] would produce diffusive detachment of electron streamtubes from the magnetic field. In yet another scenario, the plasma-induced magnetic field would stretch the MN axially to infinity, thereby circumventing the problem associated with closed-field lines [19].

We have recently shown that these mechanisms do not lead to *inward* detachment in a propulsive MN; quite on the contrary, electron diffusion detaches electron tubes *outward* from the magnetic lines, and the magnetic field induced by diamagnetic plasma currents opposes the applied one, thus increasing the divergence of the resulting MN instead of decreasing it [20]. This disagreement originates from two central limitations in previous models. Firstly, the detachment problem is studied independently and separately from the thrust production problem, in particular by neglecting the plasma thermal energy. In fact, a cold plasma behaves paramagnetically with respect to the applied field, instead of diamagnetically as expected in a hot plasma. This alters both the plasma acceleration and detachment processes. Secondly, these models assume local current ambipolarity (LCA), that is $\tilde{\jmath} = \mathbf{0}$ everywhere (with $\tilde{\jmath}$ the longitudinal electric current density of the plasma), forcing ion and electron streamtubes to coincide everywhere.

However, LCA is doubly unsatisfactory. Mathematically, LCA is a vector condition supplanting the *scalar* one, $\nabla \cdot \tilde{\jmath} = 0$, thus leading to an incompatibility. Physically, LCA does not describe the actual (and useful) expansion regime in a propulsive MN, in which the light electrons are highly magnetized and follow faithfully the magnetic tubes, but the heavy ions are essentially unmagnetized and governed by the ambipolar electric field of the plasma, $-\nabla\phi$. This means that, unless $-\nabla\phi$ is *precisely* the value required by ions to expand along electron tubes, they will separate from magnetic tubes. Indeed, our previous studies on the near-region of the MN [9] showed the incipient inward separation of ions from electron/magnetic tubes, even in a fully quasineutral, globally-current-free plasma, leading naturally to $\tilde{\jmath} \neq \mathbf{0}$ already before the MN turning point.

The purpose of this paper is to demonstrate the necessity of this separation and prove that it continues downstream and becomes a dominant feature of the two-dimensional (2D) plasma expansion in the MN, enabling the detachment of most of the plasma into a free expanding plume. This phenomenon is shown to occur even for ion magnetizations greater than those expected in propulsive applications, supporting its generality. The ambipolar electric field is shown to be insufficient to sustain the LCA condition. Finally, the conventional divergence angle of the 95%-mass flux plasma tube is used to quantify the detachment efficiency.

The discussion is based on the two-fluid, 2D model of the supersonic plasma expansion in a divergent MN presented in [9], for which only a self-contained description of the assumptions and physics is given here. Let a plasma column of radius R be injected sonically at the throat ($z = 0$) of a divergent magnetic field. The plasma is nearly collisionless (as required in an efficient MN), and globally current-free, ensuring that the thruster does not accumulate electric charge with time. As in typical thruster operation regimes [3–5], it is assumed that electrons carry the internal energy of the plasma, and are fully magnetized, meaning that electron and magnetic streamtubes coincide everywhere. For the present purposes, electrons are considered isothermal and their inertia is neglected. Heavier, colder ions on the other hand can have any magnetization degree. The quasineutral ion and electron response is determined consistently without assuming LCA, by their corresponding continuity and momentum equations,

$$\nabla \cdot (n\boldsymbol{u}_\mathrm{i}) = 0; \qquad \nabla \cdot (n\boldsymbol{u}_\mathrm{e}) = 0, \tag{1}$$

$$m_\mathrm{i}(\boldsymbol{u}_\mathrm{i} \cdot \nabla)\boldsymbol{u}_\mathrm{i} = -e\nabla\phi + e\boldsymbol{u}_\mathrm{i} \times \boldsymbol{B}, \tag{2}$$

$$\mathbf{0} = -T_\mathrm{e}\nabla \ln n + e\nabla\phi - eu_{\theta e}B\mathbf{1}_\perp, \tag{3}$$

where symbols are conventional and $\mathbf{1}_\perp = \mathbf{1}_\theta \times \boldsymbol{B}/B$ is a unit vector perpendicular to the magnetic field. The resulting ion problem is accurately integrated with the method of characteristics. The prior limitation in [9] of simulating beyond the MN turning point in the simulations is overcome now by propagating the ion characteristic lines in local intrinsic coordinates (instead of z–r) and renormalizing the plasma density downstream.

Ion magnetization is characterized by the local ion gyrofrequency parameter, $\hat{\Omega}_\mathrm{i}(z, r) = eRB(z, r)/\sqrt{T_e m_\mathrm{i}}$. The magnetic field geometry $\boldsymbol{B}(z, r)$ is defined by the coil layout, while the coil current controls the MN strength B_0 and thus $\hat{\Omega}_{\mathrm{i}0}$, with subindex 0 referring to the origin, $(z, r) = (0, 0)$. The simulations shown here use for illustration the MN of a single current loop of radius $3.5R$ located at $z = 0$. To amply cover the foreseen application range ($\hat{\Omega}_{\mathrm{i}0} < \mathcal{O}(10)$ for the thrusters of [4, 5, 21–23]), two magnetic strengths $\hat{\Omega}_{\mathrm{i}0} = 1$ and 200 are used. Finally, the initial plasma density and potential profiles are $n(0, r) = n_0 J_0(a_0 r/R)$ and $\phi(0, r) = 0$, with J_0 the Bessel function of the first kind and a_0 its first zero (see figure 2 in [9]); ions and electrons enter the divergent MN axially with the sonic velocity of ions, $\sqrt{T_\mathrm{e}/m_\mathrm{i}}$. Note that the model can be fully normalized with T_e, m_i, e, R and n_0.

Plasma detachment due to ion separation in the MN far-region is illustrated in figure 1, which shows the integrated ion (i.e. mass) flux from the axis (0%) to the plasma–vacuum boundary (100%) and the ion streamtubes containing 50% and 95% of the ion flux, for the two simulated cases. Except for the 100% tube, where *global* current ambipolarity and quasineutrality ensure that ion and electron streamtubes coincide, ion tubes detach progressively from their initially matching magnetic/electron tubes, becoming straight downstream and not flowing back; only a marginal fraction of the ion flow (less than 1%) is actually deflected backward. (Indeed, this backflow fraction is not larger than the one found in ion or Hall thrusters.) The ion detachment observed in the MN near-region (say, for $z/R < 15$) agrees with the existing vacuum-chamber measurements [5, 21–24] (see e.g. figure 4 of [21] and figure 5 of [24]). A comparison of figures 1(*a*) and (*b*) shows that the same detachment mechanism operates at any $\hat{\Omega}_{\mathrm{i}0}$, with only a minor increase in jet divergence at high $\hat{\Omega}_{\mathrm{i}0}$.

A quantitative assessment of the detachment efficiency can be carried out using the effective divergence of the resulting

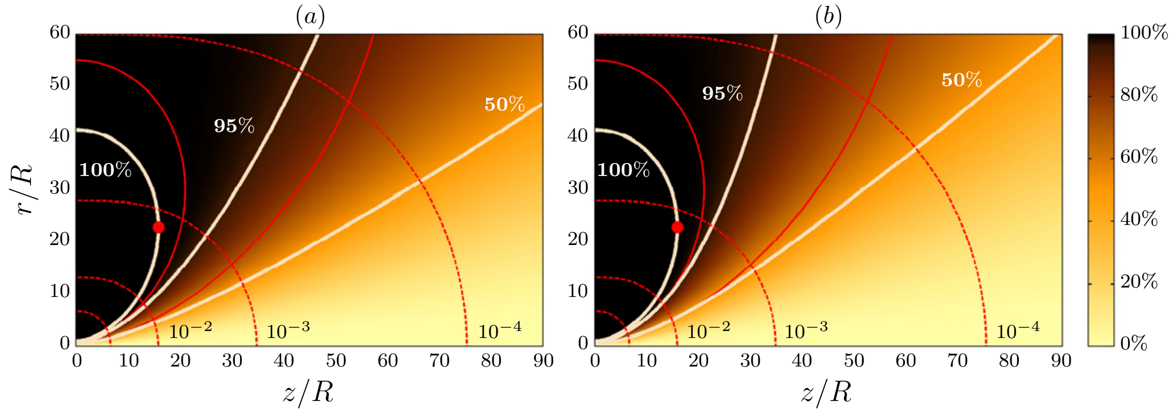

Figure 1. Ion inward detachment for two magnetic strengths: (*a*) $\hat{\Omega}_{i0} = 1$ and (*b*) 200. Thick solid white lines are ion streamtubes containing 50, 95 and 100% of the plasma mass flow. The thin solid red lines show the initially corresponding magnetic streamtubes. 2D map shows the integrated mass flow (0% on the axis, 100% on the beam edge). Dashed red lines are $B/B_0 = $ const lines. The red dot marks the MN 'turning point'.

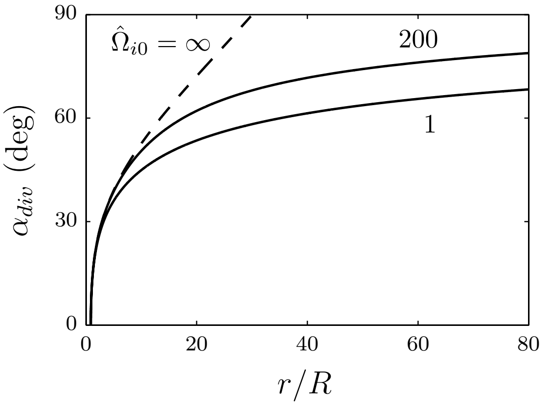

Figure 2. Evolution, for different $\hat{\Omega}_{i0}$, of the divergence angle of the 95%-mass flow tube versus the radius of that tube. The dashed line corresponds to the total ion-magnetization limit, $\hat{\Omega}_{i0} = \infty$.

plasma plume, which is conventionally defined as the angle $\alpha_{\rm div}$ formed between the 95%-mass flow streamtube and the plume centerline. Figure 2 plots the spatial evolution of $\alpha_{\rm div}$ for the two simulations of figure 1 and for the full ion-magnetization limit, $\hat{\Omega}_{\rm i} = \infty$, in which ion detachment would be null everywhere. Clearly, the larger $\hat{\Omega}_{\rm i0}$ is, the farther out ion detachment starts to be apparent, and the larger the resulting plume divergence is. Interestingly, $\alpha_{\rm div}$ does not present an asymptote in the simulated region, and continues to increase with a decreasing rate. This behavior is related to the residual pressure in the plasma downstream, and is also observed in unmagnetized, isothermal plasma plumes [25]. Nevertheless, the most striking result is the modest difference between the 'near unmagnetized' case ($\hat{\Omega}_{\rm i0} = 1$) and the 'high magnetization' case ($\hat{\Omega}_{\rm i0} = 200$), which, in spite of the large $\hat{\Omega}_{\rm i0}$, is still far from the $\hat{\Omega}_{\rm i0} = \infty$ case.

We now turn to analyze the physics behind the 2D detachment of the plasma jet. Firstly, for the *massless, confined* electrons, momentum equation (3) states that (i) on each magnetic/electron tube, the electric potential $\phi(z, r)$ and plasma density n are related by a tube-dependent Boltzmann relation and (ii) in the direction of $\mathbf{1}_\perp$ the expanding pressure force component $f_{\rm p} = -T_{\rm e}\partial \ln n/\partial \mathbf{1}_\perp$ is balanced by the confining electric and magnetic forces, $f_{\rm e} = e\partial\phi/\partial\mathbf{1}_\perp$ and $f_{\rm m} = eu_{\theta e}B$. Figure 3(*a*) quantifies the relative strength of these three force components. Observe that $f_{\rm m} \approx f_{\rm p} \sim T_{\rm e}/R \gg f_{\rm e}$ near the throat, while downstream f_e gradually takes over the electron confinement task and $f_{\rm m}$ diminishes (except at the plasma–vacuum edge, where $f_{\rm m}$ remains large). In the bulk of the plasma, $f_{\rm p}$ decreases gradually downstream as can be inferred from the behavior of plasma density n, shown in figure 3(*b*).

Secondly, for the ions, the shape of the streamtubes is dictacted by the projection of equation (2) along $\mathbf{1}_{\perp i}$, i.e. the meridional unit vector perpendicular to ion velocity (note that $\mathbf{1}_{\perp i} \neq \mathbf{1}_\perp$ in general),

$$m_{\rm i}\kappa_{\rm i}(u_{zi}^2 + u_{ri}^2) = -e\partial\phi/\partial\mathbf{1}_{\perp i} + \mathbf{1}_\perp \cdot \mathbf{1}_{\perp i} e u_{\theta i} B + \mathbf{1}_r \cdot \mathbf{1}_{\perp i} m_{\rm i} u_{\theta i}^2/r, \tag{4}$$

where $\kappa_{\rm i}$ is the meridional curvature of ion streamtubes. From the conservation of ion canonical angular momentum, stated by the θ component of equation (2), $rm_{\rm i}u_{\theta i} + e\int_0^r B_z r{\rm d}r$ is a constant on each ion streamtube. Hence, for initially non-rotating ions, $u_{\theta i} \leqslant \hat{\Omega}_{\rm i0}\sqrt{T_{\rm e}/m_{\rm i}}$. Thus, a rough, conservative estimate of $\kappa_{\rm i}$ in equation (4) with $e\partial\phi/\partial\mathbf{1}_{\perp i} \sim f_{\rm e} \sim T_{\rm e}/R$ shows that

$$\kappa_{\rm i} R \sim (1 + \hat{\Omega}_{\rm i0}\hat{\Omega}_{\rm i} + \hat{\Omega}_{\rm i0}^2 R/r)/M^2, \tag{5}$$

where $M^2 = m_{\rm i}(u_{zi}^2 + u_{ri}^2)/T_{\rm e}$ is the (squared) ion Mach number. When ions are unmagnetized, as in the case of propulsive interest, $\kappa_{\rm i}$ is dominated by the electric force, while the magnetic and centrifugal terms are negligible, so $\kappa_{\rm i} R \sim 1/M^2$ vanishes as ions are accelerated. For $\hat{\Omega}_{\rm i0}$ large, a *paramagnetic* ion magnetic force develops, which tries to restore ion separation, with two negative effects: axial ion deceleration and increment of plume divergence. The crucial point is that (except in the $\hat{\Omega}_{\rm i0} \to \infty$ limit) only the electric force remains downstream once ions demagnetize, but it alone is insufficient to keep ion tubes matched with their corresponding electron/magnetic tubes as M increases, and eventually $\kappa_{\rm i} R \to 0$, ensuring the escape of most of the ion

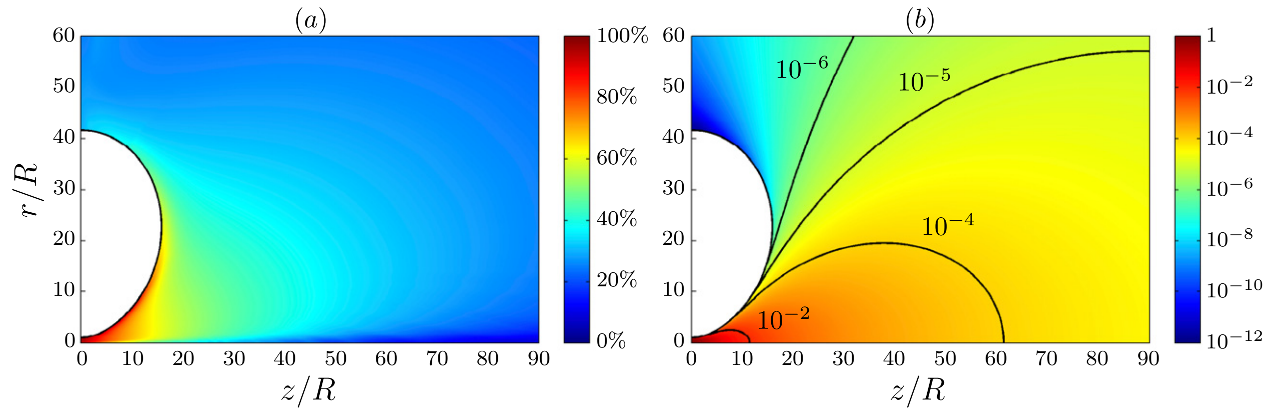

Figure 3. Maps for the $\hat{\Omega}_{i0} = 1$ case of (*a*) perpendicular magnetic-to-pressure force ratio on electrons, $f_m/f_p (= 1 - f_e/f_p)$, and (*b*) normalized plasma density, n/n_0.

flow from the MN as confirmed by the simulations of figures 1 and 2. Therefore, inward ion separation (and the formation of longitudinal currents) is an unavoidable, beneficial detachment phenomenon in a propulsive MN.

Observe that ion separation does not necessarily imply the formation of non-neutral regions in the plume, nor the existence of a net electric current from the plasma source. However, the separation of ion and electron 2D tubes is intimately linked to the formation of local longitudinal electric currents $\tilde{\jmath}$. Had the LCA condition been imposed, ion separation would be impeded, and detachment consequently concealed. Moreover, while in our model equation (3) determines ϕ and equation (4) determines κ_i, LCA forces $\kappa_i \equiv \kappa_B$ (the magnetic curvature) and ϕ is then determined again by equation (4), which is the incompatibility commented above (and clearly overestimates the electric field, in view of the actual ion streamlines).

It is important to notice that the existence of longitudinal currents does not affect the 2D ion response or the MN operation (at least in the small plasma beta regime). Indeed, the parallel electron drift velocity $u_{\parallel e}$ does not intervene in equations (2) and (3). Therefore, the determination of $\tilde{\jmath}$ is uncoupled from the rest of the plasma problem [9] (and in particular, from the determination of κ_i), which reflects the fact that electrons respond globally, not locally, to the expansion. Nevertheless, these currents must find a closure, both upstream (within the plasma source) and downstream. While the details of this closure are beyond the capabilities of the present model, we can still conjecture about the physical mechanisms that will allow it. Firstly, inside the plasma source, density and collisionality are large, allowing cross-field diffusion to close moderate electric currents. Secondly, in the large downstream region, currents will be canceled by the combination of multiple processes, such as geometrical decay of current densities and electron demagnetization. Note, however, that electron demagnetization will take place much further downstream than for ions, given the large mass difference ($m_i/m_e \sim 10^5$ for usual propellants).

Summarizing, this analysis concludes that the separation of inertia-driven, unmagnetized ions due to the insufficiency of the ambipolar electric field in the far-region (well below the LCA value) constitutes a central physical mechanism enabling the inward detachment of the mass and momentum of a collisionless plasma jet in a propulsive MN, even in the full electron magnetization limit.

Clearly, as advanced above, our present model does not include several effects that become important near the jet edge or far downstream. We comment very briefly on some of them. Firstly, relaxation of the full-electron-magnetization condition brings up resistivity and azimuthal electron inertia effects. Likewise, a lower applied magnetic field or a larger density increases the influence of plasma-induced magnetic field effects. All three phenomena cause outward electron detachment and increase jet divergence [13, 26]. The conflicting requirements of high magnetic field (to extend the region where these effects are negligible) and low ion magnetization (to foster their downstream detachment and achieve low beam divergence) indicate the existence of an optimal magnetic strength range, which depends on the m_i/m_e ratio.

Secondly, the assumption of electron isothermality in our model, while not central for the detachment process, leads to an unphysical logarithmically infinite drop of the electric potential downstream. Limited laboratory and in-space evidence suggests that the electric potential would vanish proportionally to $n^{\gamma-1}$ with $\gamma \sim 1.1$–1.2 (as in a polytropic approximation). Kinetic approaches of related problems would suggest a *collisionless cooling* of the 2D expanding electrons based on adiabatic invariants and effective potential barriers [27–29]. Note, nonetheless, that the isothermal limit yields the worst-case scenario for detachment, since it overestimates the pressure (and thus the electric force) downstream.

Lastly, figure 3(*b*) shows that beam rarefaction is large downstream and even larger sideways. Space-charge effects growing with decaying density will eventually invalidate quasineutrality and, since they set an upper bound to the electric field, they could further limit ion tube divergence. Also, the presence of a tenuous background plasma (in the chamber or in space) would smooth out the hard plasma–vacuum edge assumed here, and dominate the electric field as jet and background densities become comparable.

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