

RELATIVE DYNAMICS AND CONTROL OF AN ION BEAM SHEPHERD SATELLITE

Claudio Bombardelli,^{*} Hodei Urrutxua,[†] Mario Merino,[‡]
Eduardo Ahedo[§] and Jesús Peláez[¶]

The ion beam shepherd (IBS) is a recently proposed concept for modifying the orbit and/or attitude of a generic orbiting body in a contactless manner, which makes it a candidate technology for active space debris removal. In this paper we deal with the problem of controlling the relative position of a shepherd satellite coorbiting at small separation distance with a target debris. After deriving the orbit relative motion equations including the effect of the ion beam perturbation we study the system stability and propose different control strategies.

INTRODUCTION

Among the different strategies proposed to actively remove space debris from low and geostationary Earth orbits, the use of a highly collimated neutralized plasma beam pointed at a generic debris from a nearby "shepherd satellite" has been proposed recently.¹ The beam would transfer enough momentum to modify the orbit and/or attitude of the debris from a safe distance in a controlled manner without the need for docking. Although in principle conceptually simple, the proposed removal approach involves new and interesting challenges from the dynamics and control point of view. Most importantly, the debris shepherd and the space debris should be simultaneously de-orbited (or re-orbited) in a controlled and reliable way, keeping a safe distance between each other and avoiding collisions. This implies not only the need for advanced sensors, actuators and control strategies, but also, and first of all, the need for accurate models describing the dynamic interactions between the debris and the ion beam. The present article studies such interactions and their implications on the relative position control problem. First we derive a simplified semi-analytical formula for computing the transmitted beam force and gradient considering a conical beam and spherical target (cannonball model). We then proceed with the relative equations of motions of the shepherd-debris system with respect to a local orbiting frame including the beam force perturbation. The relative dynamics equations are integrated numerically with the aid of an in-house custom developed simulation software. After demonstrating that the beam force does not add stability to the (already unstable) relative dynamic problem, we design a simple PD control strategy for the shepherd satellite and compute the boundary of the stable region in the control parameter space. Results show that under the assumption that an accurate estimation of the relative position is available, the proposed

^{*}Research Fellow, Space Dynamics Group, Technical University of Madrid, Madrid, E-28040, Spain.

[†]PhD candidate, Space Dynamics Group, Technical University of Madrid, Madrid, E-28040, Spain.

[‡]PhD candidate, Plasma and Space Propulsion Group, Plaza Cardenal Cisneros 3

[§]Professor, Plasma and Space Propulsion Group, Technical University of Madrid, Madrid, E-28040, Spain

[¶]Professor, Space Dynamics Group, Technical University of Madrid, Madrid, E-28040, Spain

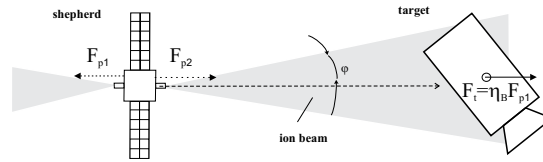


Figure 1. Schematic of ion beam shepherd satellite deorbiting a space debris

control scheme provides relative position stability for quasi-circular or mildly eccentric low Earth orbits.

THE ION BEAM SHEPHERD (IBS)

The ion beam shepherd concept (IBS) is a novel use of space electric propulsion in which the plasma accelerated by an ion thruster (or similar plasma propulsion device) is directed against the surface of a target object to exert a force (and a torque) upon the target from a distance of a few times its size (Fig.1). The force transmitted comes from the variation of momentum of the plasma ions (typically xenon) impacting against the surface of the object and penetrating its outermost layers before being stopped. This simple idea, in which the accelerated plasma is used to produce an action rather than a reaction, can be used to remotely maneuver objects in space without physical contact (docking). A promising application is found in the area of active removal of space debris, which are non-cooperating targets that can be extremely difficult to dock to as they can exhibit chaotic attitude motion. During a typical LEO deorbiting mission the IBS would rendezvous with the target debris and, while coorbiting at constant distance in front of the latter, have one of its ion beams constantly pointed at its surface to produce a small continuous drag force able to reduce the orbit semimajor axis by a few hundreds of kilometers in a few weeks or months.¹

BEAM-TARGET INTERACTION: A “CANNONBALL MODEL”

The physical interaction between a solid body and an ion-beam is described in details in a previous reference.² For the purpose of the present work we here briefly recall the main parameters characterizing the interaction. The evolution of the quasi-neutral plasma plume sufficiently far from the thruster (far-field region) can be approximated with a self-similar fluid model that hinges on two main parameters, namely the upstream beam divergence angle φ and the plasma Mach number M . While the former is related to the complex evolution of the beam in the near-field region the latter is influenced by the characteristics of the neutralizing hollow cathode as well as the plasma axial velocity. In the ideal case in which $M \rightarrow \infty$ the plasma would expand as a cone of divergence $h' = \tan \varphi$, which for state-of-the-art ion engines can be as low as 10-15 degrees. In practice $M \sim 30$ making the beam expand as a "funnel" under the effect of electron pressure² so that the effective beam divergence ends up being somewhat higher depending on the debris-shepherd separation distance. Across a generic axial section in the far-filed region the beam current density follows approximately a Gaussian profile extending (theoretically) to infinity in the transverse direction. In practice, the radius of a section containing 95% of the total current of the beam is commonly taken as the local width of the beam.

A compact semi-analytical expression of the force transmitted to a target in beam space is here derived by considering a perfectly spherical target. In this way the force is completely determined once the sphere size and position with respect to the beam is known.

Beam force and momentum transfer efficiency

Let us consider a spherical target of radius R_S whose geometrical center is located at a distance ρ from the beam source with an angular offset α from the axis of the beam. Let $\langle x, y, z \rangle$ be a Cartesian reference system centered at the beam cone vertex with x along the beam axis, z normal to the plane containing the beam axis and the sphere center and y following the right-hand rule (Figure 2). Under the assumption of a perfectly conical beam of divergence h' the three-dimensional density distribution (here assumed Gaussian) reads:

$$n = \frac{3n_0}{h'^2 x^2} \exp \left[-\frac{3(y^2 + z^2)}{h'^2 x^2} \right] \quad (1)$$

where n_0 is the average plasma density over the cross section $\mathcal{D} : (y^2 + z^2) \leq 1$ containing the total beam current. The cross section \mathcal{D} can be thought of as the thruster exit cross section.

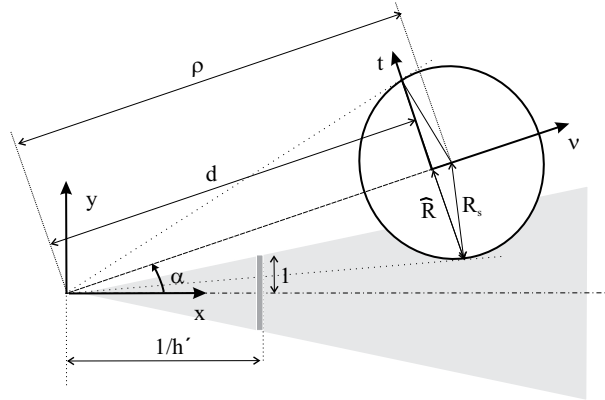


Figure 2. Beam-target interaction geometry

The velocity of the conical beam, whose axial component u_0 is here assumed constant, obeys:

$$\mathbf{u} = \left(u_0, \frac{y}{x}, \frac{z}{x} \right)^T \quad (2)$$

The force transmitted by the beam can be computed with the linear momentum equation applied to the spherical cap intercepted by the beam (Figure 2):

$$\mathbf{F} = \int_{\Sigma} m_i n \mathbf{u} (\boldsymbol{\nu}^T \mathbf{u}) dS \quad (3)$$

In the above equation m_i is the ion mass, Σ is the base of the spherical cap and

$$\boldsymbol{\nu} = (\cos \alpha, \sin \alpha, 0)^T$$

the unit vector normal to Σ . The force \mathbf{F} can be projected along $\boldsymbol{\nu}$ and along the in-plane tangential direction to Σ yielding, respectively:

$$F_\nu = \mathbf{F}^T (\cos \alpha, \sin \alpha, 0)^T \quad (4)$$

$$F_t = \mathbf{F}^T (-\sin \alpha, \cos \alpha, 0)^T \quad (5)$$

The coordinate of a generic point P of Σ can be written as:

$$x_P = d \cos \alpha - r \sin \alpha \cos \phi \quad (6)$$

$$y_P = d \sin \alpha + r \cos \alpha \cos \phi \quad (7)$$

$$z_P = r \sin \phi \quad (8)$$

where d is the distance of Σ from the beam source, which is related to the sphere center distance ρ and radius R by the identity:

$$d = \rho \left(1 - \frac{R^2}{\rho^2} \right), \quad (9)$$

while $(r, \phi)^T$ is the position of P relative to the center of Σ in polar coordinates.

The plasma density at P obeys:

$$n = \frac{3n_0 g(P)}{h'^2 x_P^2}, \quad (10)$$

with:

$$g(P) = \exp \left[-\frac{3(y_P^2 + z_P^2)}{h'^2 x_P^2} \right]. \quad (11)$$

After substituting Eqs. (6-8) into Eq. (10,2) and, in turns, into Eq. (3-??) and normalizing with respect to the thruster force $F_0 = \pi m_i n_0 u_0^2$ one obtains the non-dimensional forces:

$$f_\nu = \frac{F_\nu}{F_0} = \frac{3d^2}{\pi h'^2} \int_0^{2\pi} \int_0^{\hat{R}} \frac{g(P)r}{x_P^4} dr d\phi \quad (12)$$

$$f_t = \frac{F_t}{F_0} = \frac{3d}{\pi h'^2} \int_0^{2\pi} \int_0^{\hat{R}} \frac{g(P)r^2 \cos \phi}{x_P^4} dr d\phi \quad (13)$$

where:

$$\hat{R} = \sqrt{R_S^2 - \frac{R_S^4}{\rho^2}}. \quad (14)$$

Finally, the dimensionless beam force in the axial and radial direction with respect to the beam are:

$$f_x = f_\nu \cos \alpha - f_t \sin \alpha, \quad (15)$$

$$f_r = f_\nu \sin \alpha + f_t \cos \alpha. \quad (16)$$

It is convenient, for later use, to introduce the geometric parameter χ defined as the ratio between the sphere radius and the local beam radius R_B computed for an axial ($\alpha = 0$) distance from the beam source equal to ρ . For a conical beam the parameter χ reads:

$$\chi = \frac{R_S}{R_B} = \frac{R_S}{h'\rho}. \quad (17)$$

An exact analytical solution of the integrals (12,13) exists for small α . After expanding in Taylor series, solving the integrals and eliminating R_S through Eq. (17) we obtain:

$$f_x = \eta_B \left[1 + \frac{9\chi^2 (\chi^2 h'^4 - h'^2 + 2)}{2h'^2 (1 - h'^2 \chi^2)^3} \alpha^2 + \mathcal{O}(\alpha^4) \right], \quad (18)$$

$$f_r = (1 - \eta_B) \left[\frac{3\chi^2}{1 - h'^2 \chi^2} \alpha + \mathcal{O}(\alpha^3) \right], \quad (19)$$

where η_B is the beam momentum transfer efficiency, i.e. the ratio between the force transmitted to the target placed on the beam axis and the thruster force, which has the compact expression:

$$\eta_B = f_x(\alpha = 0) = 1 - \exp\left(-\frac{3\chi^2}{1 - h'^2 \chi^2}\right).$$

Beam gradient

Of interest for the dynamics and control of the target-shepherd relative position is the dimensionless beam gradient matrix. The latter is here defined as the dimensionless gradient of the beam force vector when the target center of mass lays on a nominal equilibrium position \mathbf{r}_{eq} :

$$\mathbf{B} = \frac{R_B}{F_0} \left(\frac{d\mathbf{F}}{d\mathbf{r}} \right)_{\mathbf{r}=\mathbf{r}_{eq}}.$$

In the above equation the thruster force F_0 and the local beam radius R_B are taken as reference quantities for the non-dimensionalization.

For a sphere whose center of mass and geometrical center coincide a nominal equilibrium configuration is typically found when the sphere center lays on the beam axis. Using Eqs. (17-19) we have that for $\alpha = 0$:

$$dx = d\rho = -\frac{R_B}{h'\chi}d\chi,$$

$$dy = dz = \rho d\alpha = \frac{R_B}{h'}d\alpha,$$

which can be employed to finally derive the beam gradient matrix as:

$$\mathbf{B} = \begin{bmatrix} -2b & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & b \end{bmatrix},$$

with:

$$b = \frac{3h'\chi^2}{(1-h'^2\chi^2)^2} \exp\left(-\frac{3\chi^2}{1-h'^2\chi^2}\right) \geq 0.$$

The function $b(\chi)$ is zero for $\chi = 0$ and $\chi \rightarrow \infty$ and has a maximum:

$$b_{max} \simeq h'/e,$$

corresponding to:

$$\chi_{max} \simeq \frac{\sqrt{3}}{3}$$

RELATIVE DYNAMICS AND CONTROL

After linearizing the local gravitational field around the shepherd orbital position the equations of motion governing the evolution of the debris relative position ρ with respect to the shepherd in a local Frenet frame are:

$$\ddot{\rho} + \left(\Omega\Omega + \dot{\Omega} - \mathbf{G}\right)\rho + 2\Omega\dot{\rho} = \frac{\mathbf{F}_T}{m_T} - \frac{\mathbf{F}_S}{m_S}, \quad (20)$$

where Ω represent the angular velocity matrix, \mathbf{G} is the gravity gradient matrix and \mathbf{F}_T , \mathbf{F}_S are the force vectors acting on the target (T) and shepherd (S), respectively.

Eqs. (20) need to be accompanied by the shepherd orbit evolution written in inertial axes:

$$\ddot{\mathbf{r}}_S = -\mu \frac{\mathbf{r}_S}{r_S^3} + \frac{\mathbf{F}_S}{m_S},$$

where \mathbf{r}_S is the shepherd center of mass geocentric inertial position and μ the Earth gravitational parameter.

Open-loop stability in quasi-circular orbit

In order to gain insight into the complex relative dynamics of the IBS-target system it is convenient to start off with the simplest possible model. To this end we introduce the following assumptions:

1. The target is modeled as a sphere whose center of mass and geometrical center coincide.
2. The orbit of the IBS-target system is considered quasi-circular
3. The ion beam is assumed conical and constantly pointed along the shepherd instantaneous velocity vector, has constant intensity, and transmits to the debris a force, which only depends on the debris center of mass location relative to the shepherd. Additional thrusters in the three directions can be used to control the position of the IBS but do not affect the target dynamics (i.e. they are not pointed against the target).
4. All external perturbations are neglected with the exception of the ion beam force.

If the target orbit evolves in a quasi-circular manner Eqs. (20) can be well approximated by the perturbed Clohessy-Wiltshire equations. With respect to a reference frame with the y axis along the instantaneous velocity vector and z along the instantaneous angular momentum vector we have:

$$\begin{cases} \ddot{x} - 2\omega\dot{y} - 3\omega^2x = F_{xT}/m_T - F_{xS}/m_S \\ \ddot{y} + 2\omega\dot{x} = F_{yT}/m_T - F_{yS}/m_S \\ \ddot{z} + \omega^2z = F_{zT}/m_T - F_{zS}/m_S \end{cases} \quad (21)$$

where ω is the time-varying orbit mean motion. In the above equations the Euler acceleration terms have been neglected since $\dot{\omega} \ll \omega^2$ for slow deorbiting.

Among the possible equilibrium configurations satisfying Eqs. (5) the most effective for deorbiting and reorbiting operations is the one in which the IBS and the debris are coorbiting at close distance so that the nominal relative position vector reads:

$$\bar{\boldsymbol{\rho}} = (0, \quad \rho, \quad 0)^T. \quad (22)$$

After substituting Eq. (22) into Eqs. (21) one obtains the nominal “zero-drift” force vectors:

$$\bar{\mathbf{F}}_T = (0, \quad F_T, \quad 0)^T, \quad (23)$$

$$\bar{\mathbf{F}}_S = \left(0, \quad \frac{m_S}{m_T} F_T, \quad 0 \right)^T. \quad (24)$$

For the purpose of investigating the system open loop stability the control force on the shepherd is kept constant and equal to the nominal value computed above while the force transmitted by the beam to the target is linearized along its nominal value to yield:

$$\mathbf{F}_T \simeq \bar{\mathbf{F}}_T + \frac{F_0}{R_B} \mathbf{B} (\boldsymbol{\rho} - \bar{\boldsymbol{\rho}}),$$

or, equivalently:

$$\begin{aligned}
F_{Tx} &\simeq \frac{bF_0}{R_B}x \\
F_{Ty} &\simeq -\frac{2bF_0}{R_B}(y - \rho) \\
F_{Tz} &\simeq \frac{bF_0}{R_B}z,
\end{aligned} \tag{25}$$

which substituted into Eqs. (5) together with Eq. (7) yield:

$$\begin{aligned}
\ddot{x} - 2\omega\dot{y} - 3\omega^2x &= \frac{bF_0}{R_B}x \\
\ddot{y} + 2\omega\dot{x} &= -\frac{2bF_0}{R_B}(y - \rho) \\
\ddot{z} + \omega^2z &= \frac{bF_0}{R_B}z.
\end{aligned} \tag{26}$$

After introducing the dimensionless target offset components:

$$\delta_x = \frac{x}{R_B}; \quad \delta_y = \frac{y - \rho}{R_B}; \quad \delta_z = \frac{z}{R_B},$$

and the dimensionless time:

$$\tau = \omega t,$$

Eqs. (26) can be rewritten as:

$$\begin{cases} \delta_x'' - 2\delta_y' - (3 + \gamma)\delta_x = 0 \\ \delta_y'' + 2\delta_x' - 2\gamma\delta_y = 0 \\ \delta_z'' + (1 - \gamma)\delta_z = 0 \end{cases} \tag{27}$$

where primes indicate derivatives with respect to the dimensionless time τ and where we have introduced the dimensionless stiffness coefficient of the beam-target interaction:

$$\gamma = \frac{bF_0}{m_T\omega^2R_B} > 0. \tag{28}$$

Eqs. (27) can be rewritten in compact form as:

$$\mathbf{s}'' = \mathbf{A}\mathbf{s},$$

where:

$$\mathbf{s} = (\delta_x, \delta_y, \delta_z, \delta_x', \delta_y', \delta_z')^T,$$

and

$$\mathbf{A} = \begin{bmatrix} 0 & \mathbf{I} \\ -\mathbf{K} & \mathbf{C} \end{bmatrix}.$$

In the above expression $\mathbf{0}$ and \mathbf{I} are the 3x3 zero and identity matrix, respectively, while:

$$\mathbf{K} = \begin{bmatrix} -3 - \gamma & 0 & 0 \\ 0 & 2\gamma & 0 \\ 0 & 0 & 1 - \gamma \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} 0 & -2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The corresponding characteristic polynomial reads:

$$P(\lambda) = P_{out}(\lambda)P_{in}(\lambda),$$

with

$$P_{out}(\lambda) = \lambda^2 + 1 - \gamma,$$

$$P_{in}(\lambda) = \lambda^4 + (1 + \gamma)\lambda^2 - 2\gamma(3 + \gamma).$$

Owing to the problem symmetry the in- and out-of-plane dynamics are decoupled. The latter remain stable if:

$$\gamma < 1,$$

which means the radial “destabilizing” effect of the beam should not prevail against the gravity gradient restoring force in the direction normal to the orbit plane. On the other hand, as it can be readily verified given the constraint 28, the open-loop in-plane dynamics are always unstable.

Control strategy

For the purpose of the present work we will make the main simplifying assumption that the relative position between the debris and the shepherd center of mass can be estimated at all times with no error. While the assumption is clearly not realistic, as the relative position measurements are a critical aspect of the concept, it allows to focus directly on the control part of the problem. Future studies will add realistic models of the different sensors employed in the guidance and navigation part, which is beyond the scope of the present study. As far as the dynamical model employed we will consider it fully deterministic and obeying to the previously derived equations.

The most straightforward way to control the IBS-debris relative motion around the nominal equilibrium configuration is to employ a three-axis thruster-based feedback control system that acts on

the shepherd according to a measured position and velocity deviation with respect to the nominal equilibrium configuration. In this way the new \mathbf{K} and \mathbf{C} matrices become:

$$\tilde{\mathbf{K}} = \begin{bmatrix} -3 - \gamma + \gamma_R & 0 & 0 \\ 0 & 2\gamma + \gamma_V & 0 \\ 0 & 0 & 1 - \gamma + \gamma_H \end{bmatrix},$$

$$\tilde{\mathbf{C}} = \begin{bmatrix} \sigma_R & -2 & 0 \\ 2 & \sigma_V & 0 \\ 0 & 0 & \sigma_H \end{bmatrix}.$$

where the R-bar (radial direction), V-bar (tangential) and H-bar (plane normal) dimensionless control gains corresponding to the control forces F_x, F_y, F_z and are defined as:

$$\gamma_R = \frac{F_x}{m_T \omega^2 R_B \delta_x}; \quad \gamma_V = \frac{F_y}{m_T \omega^2 R_B \delta_y}; \quad \gamma_H = \frac{F_z}{m_T \omega^2 R_B \delta_z},$$

$$\sigma_R = \frac{F_x}{m_T \omega^2 R_B \delta'_x}; \quad \sigma_V = \frac{F_y}{m_T \omega^2 R_B \delta'_y}; \quad \sigma_H = \frac{F_z}{m_T \omega^2 R_B \delta'_z}.$$

After computing the new characteristic polynomial one obtains the stability conditions (to be simultaneously satisfied) for the out-of-plane dynamics:

$$\sigma_H \geq 0; \quad \gamma_H \geq \gamma - 1,$$

and for the in-plane dynamics:

$$\gamma_R \geq 3 + \gamma; \quad \gamma_V \geq -2\gamma; \quad \sigma_V \geq 0; \quad \sigma_R \geq 0.$$

Now that the boundary of the stable region are defined one needs to choose proper values of the different dimensionless gains based on some optimization criterion. Instead of following the common procedure of linear quadratic regulator (LQR) control design we can opt for a more straightforward pole-placing approach in which the oscillatory part of the linear response is set to zero. To this end we impose:

$$P_{in}(\lambda) = (\lambda + m^2)^4 \quad m \in \mathbb{R},$$

which provides:

$$\begin{aligned} \gamma_V &= -\gamma + m^4 \\ \gamma_R &= 3 + \gamma + m^4 \\ \sigma_V &= -2 + 2m^2 \\ \sigma_R &= 2 + 2m^2. \end{aligned} \tag{29}$$

A similar approach can be used for the out-of-plane part providing:

$$\begin{aligned}\gamma_H &= \gamma_r - 1 + m^4, \\ \sigma_H &= 2m^2.\end{aligned}\tag{30}$$

Numerical Simulations

The performance of the proposed control system has been tested numerically with a realistic mission scenario. The debris used for the numerical simulations reported here is a spherical aluminum shell of 1.5 ton and 2 m radius acted upon by a beam of 100 mN maximum force and 10 degrees initial divergence. The nominal control position is set to 10 m, corresponding to a nominal geometric parameter $\chi \sim 0.65$. For all the simulations the J2 perturbation was, together with the ion beam, the only one acting on the system. We took as baseline a circular orbit of 82 degrees inclination and 1000 km altitude. The shepherd spacecraft mass was set to 300 kg.

Figures 3 and 4 summarize the results of the control effort after the sphere is a given position offset in the xy plane to bring it away from the nominal equilibrium condition. The case of a circular (Figure 3) and elliptic (4) orbit are considered. The PD control coefficient m has been set equal to 2.

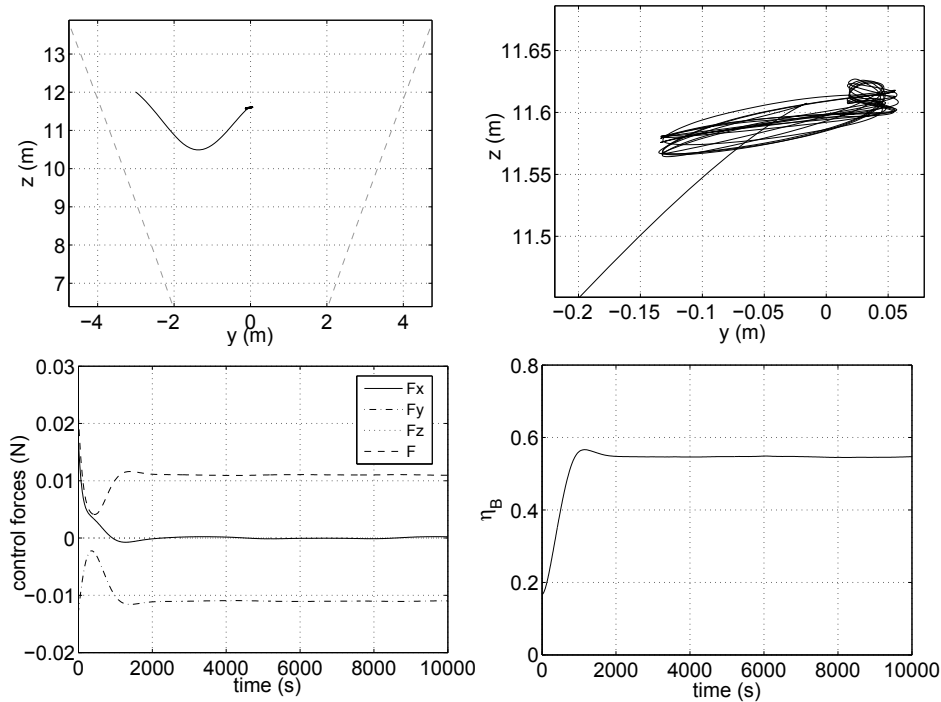


Figure 3. Spherical debris controlled trajectory with respect to the IBS (top left), trajectory close-up (top right), IBS three-axis control forces (lower left) and momentum transfer efficiency (lower right). A circular is considered.

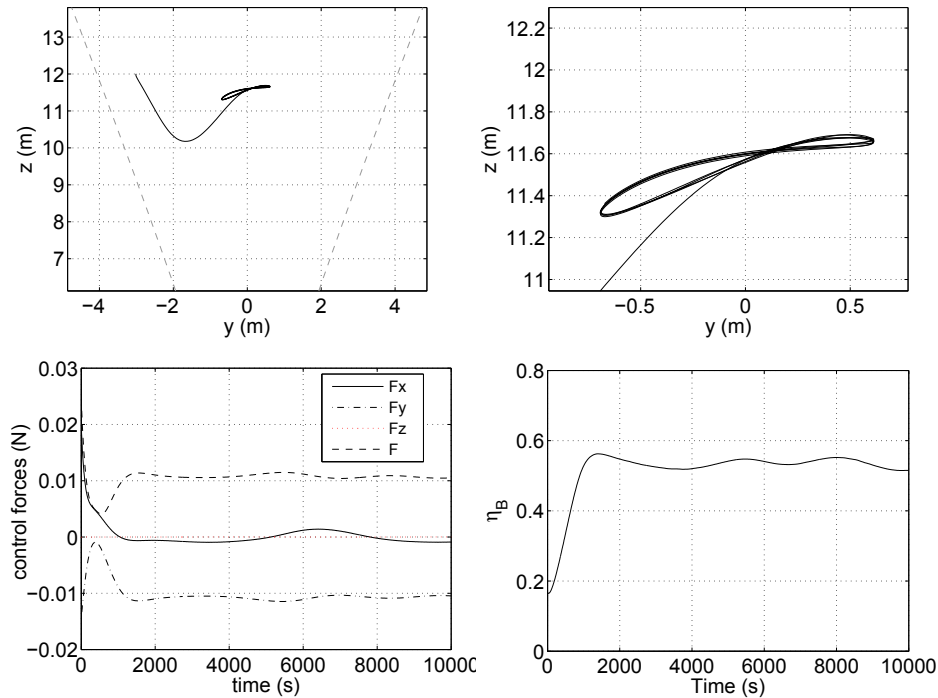


Figure 4. Spherical debris controlled trajectory with respect to the IBS (top left), trajectory close-up (top right), IBS three-axis control forces (lower left) and momentum transfer efficiency (lower right). An elliptic orbit with eccentricity 0.05 is considered.

CONCLUSIONS

The relative stability and control of a spherical target under the action of an ion beam has been investigated. Semi-analytical relations were provided that can be used for the evaluation of the beam force transmitted to a spherical target in a generic position with respect to a conical beam. Modified beam-perturbed Clohessy-Wiltshire equations for the relative motion of the IBS-target system in circular orbit were derived. Finally the stability of the relative motion was investigated showing that in the open-loop case the beam action further destabilize the already unstable relative dynamics. A properly tuned proportional-derivative control system is seen to be capable of controlling the relative motion even under non-nominal conditions assuming a perfect estimation of the relative position vector. Future studies will need to be carried out to investigate the more general case in which the latter assumption is removed.

ACKNOWLEDGMENTS

The work has been supported by the "ARIADNA Call for Ideas on Active Debris Removal" established by the Advanced Concepts Team of the European Space Agency and by the research project "Dynamic Simulation of Complex Space Systems" supported by the Dirección General de Investigación of the Spanish Ministry of Education and Science through contract AYA2010- 18796.

REFERENCES

- [1] C. Bombardelli and J. Peláez, "Ion beam shepherd for contactless space debris removal," *Journal of Guidance, Control, and Dynamics*, Vol. 34, No. 3, 2011, pp. 916–920.
- [2] M. Merino, E. Ahedo, C. Bombardelli, H. Urrutxua, and J. Pelaez, "Hypersonic Plasma Plume Expansion in Space," *IEPC-2011-086. Presented at the 32nd International Electric Propulsion Conference, Wiesbaden, Germany.*